ESSAYS ON THE USE OF THE REAL OPTIONS APPROACH

IN CONSTRUCTION PROJECTS AND BUILD-OWN-TRANSFER PROJECTS

por

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TESE DE DOUTORAMENTO EM CIÊNCIAS EMPRESARIAIS

Orientada por:

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Deste modo ou daquele modo.

Conforme calha ou não calha.

Podendo às vezes dizer o que penso,

E outras vezes dizendo-o mal e com misturas,

Vou escrevendo os meus versos sem querer,

Como se escrever não fosse uma cousa feita de gestos,

Como se escrever fosse uma cousa que me acontecesse

Como dar-me o sol de fora.

Procuro dizer o que sinto

Sem pensar em que o sinto.

Procuro encostar as palavras à idéia

E não precisar dum corredor

Do pensamento para as palavras

Nem sempre consigo sentir o que sei que devo sentir.

O meu pensamento só muito devagar atravessa o rio a nado

Porque lhe pesa o fato que os homens o fizeram usar.

Procuro despir-me do que aprendi,

Procuro esquecer-me do modo de lembrar que me ensinaram,

E raspar a tinta com que me pintaram os sentidos,

Desencaixotar as minhas emoções verdadeiras,

Desembrulhar-me e ser eu, não Alberto Caeiro,

Mas um animal humano que a Natureza produziu.

E assim escrevo, querendo sentir a Natureza, nem sequer como um homem,

Mas como quem sente a Natureza, e mais nada.

E assim escrevo, ora bem ora mal,

Ora acertando com o que quero dizer ora errando,

Caindo aqui, levantando-me acolá,

Mas indo sempre no meu caminho como um cego teimoso.

Ainda assim, sou alguém.

Sou o Descobridor da Natureza.

Sou o Argonauta das sensações verdadeiras.

Trago ao Universo um novo Universo

Porque trago ao Universo ele-próprio.

Isto sinto e isto escrevo

Perfeitamente sabedor e sem que não veja

Que são cinco horas do amanhecer

E que o sol, que ainda não mostrou a cabeça

Por cima do muro do horizonte.

Ainda assim já se lhe vêem as pontas dos dedos

Agarrando o cimo do muro

Do horizonte cheio de montes baixos.

Alberto Caeiro

À memória da minha mãe, Margarida.

À memória do meu irmão, Eduardo.

Nota Biográfica

João Adelino Neves Pereira Ribeiro nasceu no dia 25 de Setembro de 1967, em Guimarães.

Em Outubro de 1990 completou a licenciatura em Gestão de Empresas, na Universidade Portucalense Infante D. Henrique, com média final de 15 valores. Em Janeiro de 1992 terminou o programa de mestrado "Master of Business of Admnistration" (M.B.A.) pela Universidade de Birmingham, Reino Unido, com a apresentação e aprovação da respectiva dissertação final, designada de "Capital Structure and Sources of Funds with Particular Reference to Portuguese Industrial Companies". Em Novembro de 1996 obteve, por equivalência, o grau de mestre em Finanças pela Universidade Portucalense Infante D. Henrique.

Iniciou a sua actividade profissional no extinto Banco Português do Atlântico (Gabinete Central de Análise Económica e Financeira da Direcção Comercial Norte) em Outubro de 1992, tendo deixado de colaborar com esta instituição em Maio de 1993. No ano lectivo de 1992/93, iniciou a sua carreira académica, tendo assumido as funções de docente no Instituto Superior da Maia, onde leccionou, até 1994, as disciplinas de "Cálculo Financeiro" e de "Organização e Gestão de Empresas". Na Universidade Portucalense Infante D. Henrique e respectivos Institutos Politécnicos, leccionou, desde o ano lectivo de 1993/94 até ao ano lectivo de 2004/05, as disciplinas de "Cálculo Financeiro", "Contabilidade Geral", "Análise Financeira", "Auditoria Contabilística" e "Avaliação de Projectos de Investimento", aos cursos de Contabilidade, Gestão de Empresas e de Administração Pública.

Exerceu funções de Direcção Financeira em duas empresas: "Galler Portuguesa - Fábrica de Malhas, Lda.", subsidiária do grupo multinacional inglês "Courtaulds Textiles, PLC", entre Outubro de 1994 e Abril de 1996 e, entre Junho de 1996 e Abril de 2005 em "Cari - SA", empresa dedicada ao sector da construção civil e obras públicas. Foi também responsável, enquanto consultor, pela elaboração de diferentes estudos, com especial destaque para a execução de estudos de viabilidade económica e financeira no âmbito de candidaturas a diferentes programas de incentivos ao investimento produtivo, actividade que desenvolveu entre os anos de 1992 e 1994 e, mais tarde, nos anos de 2008 e 2009.

Em Outubro de 2009 iniciou o programa de Doutoramento em Ciências Empresariais na Faculdade de Economia da Universidade do Porto. Completou, em Fevereiro de 2011, a componente escolar do programa com média final de 15 valores. Iniciou, então, os trabalhos conducentes à elaboração da presente tese de doutoramento. Em 2012, apresentou o primeiro dos artigos que integra esta dissertação em duas Conferências: na 7ª edição da "Portuguese Network Finance Conference", realizada em Julho desse ano na Universidade de Aveiro e ainda na 6ª edição da "Portuguese Economic Journal Conference", realizada no mesmo mês de Julho, na Faculdade de Economia da Universidade do Porto.

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I address my first words to my supervisors, Professor Paulo Pereira and Professor Elísio Brandão.

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Professor Elísio Brandão has encouraged me so much, since the first time we talk about my possible decision of enrolling in this PhD program. From then onwards, Professor Elísio Brandão has always addressed me words of encouragement, and his example as an outstanding academic and researcher has been a source of inspiration until the present day. And I am sure that his example will keep inspiring me in the future.

I also wish to address words of deep gratitude to Professor José da Silva Costa, my first Lecturer of "Capital Budgeting and Investment Appraisal", back in the years of 1989/90. Professor José da Silva Costa outstanding skills as a lecturer and the profound commitment to his students were the reasons why I felt a deep interest for this field of knowledge, more than 20 years ago.

Likewise, I owe words of profound gratitude to Professor Manuel Oliveira Marques for his continuous encouragement and for always being available to help me in anyway I needed.

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Last - but certainly not least - I am endlessly grateful to my father, António, my brother, Casimiro José, my nephew, Casimiro António and my cousin, Isabel, for their unconditional love and support. This work also belongs to them.

Resumo

Esta tese pretende ser uma contribuição para o corpo de investigação em torno da utilização da metodologia das opções reais a projectos de construção e a projectos de construção-exploração. Pese embora estes projectos possuam uma natureza distinta da maioria dos restantes projectos de investimento, consideramos que comungam das características da irreversibilidade, incerteza e flexibilidade.

Nos Capítulos II e III, propomos dois modelos teóricos de apoio à decisão que podem ser utilizados por gestores de empresas dedicadas à realização de projectos de construção adjudicados através de um procedimento concursal competitivo. No Capítulo II, sugerimos um modelo que permite determinar o preço óptimo para a realização do projecto, baseado na avaliação de uma opção real por nós identificada e que apenas pode ser exercida pela empresa vencedora do concurso. No Capítulo III, estudamos os efeitos produzidos por um conceito específico de incerteza que rodeia os projectos de construção, e que designamos por "incerteza de volume". Com base na solução numérica apresentada no Capítulo II, propomos um modelo que permite determinar o impacto esperado que este tipo de incerteza produz no preço óptimo, através da definição de uma variável estocástica designada por "valor adicional", *i.e.*, o lucro que poderá ser gerado considerando os efeitos da incerteza referida. O modelo determina uma regra de apoio à decisão da realização de investimento de carácter incremental em capital humano e tecnologia, o qual permite quantificar o impacto esperado que a incerteza de volume causará no valor do projecto.

No Capítulo IV, no âmbito de um projecto de construção-exploração, propomos um modelo teórico, baseado na existência de duas variáveis estocásticas, cuja finalidade consiste em determinar o valor óptimo para a penalidade legal a incluir pelo governo ou outras entidades públicas na minuta do respectivo contrato, no caso de a empresa concessionária não iniciar a execução do projecto imediatamente. O modelo contempla a existência de custos sociais e considera que a entidade privada é mais eficiente do que a entidade pública a construir a infraestrutura do projecto.

Abstract

The present thesis aims to contribute to the existent body of research dedicated to the use of the real options approach in construction projects and Build-Own-Transfer (BOT) projects. Even though these type of projects have a different nature than the majority of the other investment projects, we believe that they share the same characteristics of irreversibility, uncertainty and flexibility.

In Chapters II e III, we suggest two theoretical support decision models that may be applied by construction managers in the context of bidding competitions. In Chapter II, we propose a model whose outcome is the optimal price for the execution of a construction project awarded through an appropriate bidding process. The model is based on the valuation of a specific real option, previously identified, and which can only be exercised by the selected bidder. In Chapter III, we study the effects of a particular type of uncertainty that surrounds construction projects, and which we designate as "volume uncertainty". Applying the numerical solution presented in the previous Chapter, we suggest a model that addresses the expected impact of this type of uncertainty on the project value and on the optimal bid price, through the definition of a stochastic variable, designated as "additional value", *i.e.*, the profit that may be generated through the execution of additional work. The model's outcome is the threshold value for the incremental investment in human capital and technology construction managers may undertake with the purpose of quantifying the expected impact that this type of uncertainty will produce on the project value.

In Chapter IV, we suggest a theoretical model to be applied by governments and other public entities, in the context of Build-Own-Transfer projects, based on a two-factor uncertainty approach in continuous-time, aiming to determine the optimal value for the legal penalty to be included in the contract form, in the case the selected bidder does not implement the project immediately. The model considers the existence of social costs and assumes that the private entity is more efficient than the public entity in executing the project facility.

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Chapter 1

Introduction

"Uncertainty is a quality to be cherished, therefore - if not for it,

who would dare to undertake anything?"

Auguste de Villiers de L'Isle-Adam

In the last decade, the real options approach has been increasingly adopted to address research topics concerning construction projects and Build-Own-Transfer (BOT) projects. The interest in applying the real options approach to address issues involving these types of projects is justified by the existence of high levels of uncertainty, the presence of flexibility and also recognizing that the investment costs are seldom reversible, which means that construction costs are "project-specific".

The present thesis comprises three theoretical models where the real options approach is applied to address three different research subjects. The first two models, proposed in Chapters II and III, deal with topics concerning construction projects awarded through appropriate bidding competition processes. The two models are, therefore, support decision models intended to be used by construction managers. The third model, suggested in Chapter IV, is to be applied by governments and other public entities, in the context of a BOT project, or any other contractual arrangement between a public entity and a private firm, whereby the public entity grants the construction of a facility and the operation of subsequent activities to a private entity, rather than conducting the project itself.

In Chapters II and III, the models therein suggested address practical issues construction managers have to deal during the bid preparation stage. Both models are, thus, based on the existence of a bidding competition process and may be applied either when the process is conducted by a public entity or by a private client, provided that the latter applies similar rules and procedures to those used in public contracting processes.

The model proposed in Chapter II aims to reach the optimal price that construction managers should include in their bid proposals. The model is based on the valuation of a specific real option previously identified: the option to sign the contract and invest in executing the project by the selected bidder. This implies that the selected bidder has flexibility regarding the decision of whether to sign the contract and, consequently, invest in performing the project. This flexibility does have value, as clearly stated by the option pricing theory. In fact, the construction company (or "contractor") selected by the client may decide not to sign the contract and, hence, not exercise the option to invest, at the moment the contract has to be signed. This decision may be justified by the fact that the expected amount of construction costs, which served as the basis to establish the bid price, have most likely varied from the moment the price was defined, the bid proposal delivered to the client and the day the bid results become publicly available, one of the bidders selected and invited to sign the contract. This period of time tends to be fairly long, especially in the case of large-scale projects (such as airports, high-speed railway transportation, hospitals and highways) and when a substantial number of contractors compete to win the contract. Since the bid price remains unchanged during this period, the project's expected value is probably different than the project value estimated during the bid preparation stage. In fact, the expected profit margin (or, as commonly known in construction parlance, the "mark-up bid") may now be negative. Being so, and in pure financial terms, the selected bidder should decline the invitation to sign the contract. On the contrary, if the price is higher than the expected construction costs, in the day the contract has to be signed, then the selected bidder should exercise the option to invest and sign the contract. The model is extended to accommodate the existence of penalty costs since, in some legal environments, a financial compensation may be enforced if the selected bidder refuses to enter into contract. These costs may also assume the nature of reputational costs, *i.e.*, costs that may be borne by the contractor in future bidding competitions as a result of declining the present invitation. Considering these new conditions, the selected bidder should only sign the contract and perform the project if the value of the penalty costs is greater than the difference between the price and the expected amount of construction costs, in the day the contract has to be signed.

The option identified above is only available to selected bidders, as we have mentioned. This means that, when the bid price needs to be established, *i.e.*, at the bid preparation stage, the contractor does not know if he or she is going to be selected and invited to sign the contract. In fact, a bid competition takes place and - all else equal - the project will be awarded to the contractor that presented the lowest price or, which is the same, the lowest bid. Hence, the value of the option to invest must be weighted by the probability of winning the bid. The relationship between the price (or mark-up bid) and the probability of winning the contract has been a subject of research since Friedman (1956) and Gates (1967) proposed the first two models linking these two variables. The winning function suggested in Chapter II is a two-parameter equation that respects the generally accepted inverse relationship between the mark-up bid and the probability of winning the contract. This inverse relationship between the price and the probability of winning the bid is an accepted fact both in the construction industry and in the research community. Thus, the model integrates two different components: (i) the value of the option to invest, before considering the impact of the probability of winning the bid and (ii) the probability of winning the bid. However, variations in the bid price will cause opposite effects in each of them. The value of the option to invest - before being weighted by the probability of winning the bid - increases in response to higher bid prices, whereas the probability of winning the contract decreases. Hence, the model's outcome is the result of a maximization problem, which integrates these two components and determines that, to the highest value of the option to invest, weighted by the probability of winning the contract, corresponds a specific price. Under the real options approach and considering the characteristics of the option we have identified, this price is the optimal price, and to this price corresponds the optimal mark-up bid.

In Chapter III, we turn our attention to a specific source of uncertainty surrounding construction projects, which we designate as “volume uncertainty”. Volume uncertainty is present in many construction projects since contractors do not know, during the bid preparation stage, the exact amount of work that will be executed during the construction phase. Indeed, this type of uncertainty derives from the fact that value is often hidden in the most uncertain portions of the project, as Ford et al. (2002) stated. These authors used the expression “hidden-value” to designate the value that does not exclusively result from the execution of the tasks included in the bid documents and according to the conditions therein established. Even though we acknowledge that not all hidden-value results in the execution of more volume of work, we use this expression to designate the value that is hidden and, if properly captured and quantified, may lead to the creation of additional profit through the execution of additional orders, to be placed by the client after the bidding process has ended. Thus, we may conclude that the presence of uncertainty surrounding the volume of work that will actually be executed leads to uncertainty concerning the project value. In order to assess the impact that volume uncertainty may cause on the project’s expected value, the model integrates a discrete-time stochastic variable, designated as “additional value”. This stochastic variable represents the additional profit that may be generated through the execution of more volume of work. We assume that contractors, using the skills of their own experienced staff, are able to stipulate a high-value estimate and a low-value estimate and to attribute a probability of occurrence to each of the estimates just by undertaking a preliminary analysis of the bid documents, which means that it is possible to define the statistical distribution for the values this variable may assume without the need of incurring in any incremental investment. However, in order to determine the amount of additional value, contractors often need to invest in human capital and technology and, hence, hire specialized firms and highly skilled professionals. This incremental investment needs to be undertaken for determining which of the two estimates - the high-value estimate or the low-value - is, in fact, the true value. The model’s outcome is the threshold value for the incremental investment that resolves the uncertainty involving which of the two estimates previously defined is, in fact, the true value for the additional profit. A decision rule is reached: contractors should invest in hiring external services with the purpose of determining the additional value to be generated through the execution of additional orders, placed by the client after the contract has been signed, provided that the cost of this incremental investment does not exceed the predetermined threshold value. By applying the maximization procedure suggested in Chapter II, the model also determines the optimal price that results from considering the effect of the expected additional profit, if no incremental investment is undertaken, and the optimal price in the case the incremental investment reveals that the true value for the additional profit is equal to the high-estimate or equal to the low-estimate, both previously defined.

The theoretical model proposed in Chapter IV is intended to be applied by governments and other public entities, in the context of BOT projects. We propose a contractual framework where the public entity does not impose any obligation to the the selected bidder concerning the timing to initiate the project implementation. However, this flexibility may entail a cost to the selected private firm and we suggest that this cost assumes the form of a legal penalty, which should be written down in the contract form and enforced in the case the selected bidder does not implement the project immediately. The model's outcome is the optimal value for this contractual penalty.

The method applied for determining the optimal value for the contractual penalty is based on a two-factor uncertainty approach, where both the facility construction costs and the present value of the cash flows to be generated by running the subsequent operations behave stochastically, according to geometric Brownian motions that are possibly correlated. The model considers the existence of social costs, which we define as the costs that correspond to the loss of social welfare occurring from the moment the project should have been implemented and services provided to the population and the moment the project is actually completed and services start being provided to the users. The model also considers the generally accepted argument in the literature that the private entity is more efficient than the government in constructing the project facility.

The method used comprises three different stages. First, the government decision to invest, as if the project was undertaken by the government is derived, taking into account that the government decision to invest is a function of the level of social costs the government estimates as being acceptable. Secondly, the government expectation about the private firm decision to invest, assuming that the government considers the private firm to be more efficient in constructing the facility, is derived. Since this decision is also affected by the existence of a legal penalty, we can not invoke homogeneity of degree one in the corresponding boundary conditions, which means that we can not apply the closed form solution proposed by McDonald and Siegel (1986). To overcome this problem, we follow Adkins and Paxson (2011) quasi-analytical approach, based on a set of three simultaneous equations. This set of equations enables us to reach a discriminatory boundary for the private firm, separating the waiting region from the investment region. Finally, the same system of simultaneous equations is used again in order to reach the optimal value for the contractual penalty. Since proportionality is present between the value, in absolute terms, for the optimal contractual penalty and the expected construction costs, the model determines the optimal value for the contractual penalty "per unit" of the private firm's expected construction costs.

Sensitivity analysis performed demonstrates that there is a value for the comparative efficiency factor above which there is no need to include a legal penalty in the contract, for a given level of social costs. Similarly, there is a level of social costs, for a given level of comparative efficiency, above which enforcing a contractual penalty becomes justifiable, which means that a trade-off exists between the two factors. We present the analytical solution that determine each of the threshold values. We then proceed to examine the effects, to the government, that result from including a non-optimal value for the legal penalty in the contract form. More specifically, we study the consequences of considering an inaccurate estimate for the comparative efficiency factor, and conclude that overestimating (underestimating) the selected bidder's real comparative efficiency leads to the inclusion of a below-optimal (above-optimal) value for the legal penalty in the contract form. We conclude that enforcing a non-optimal contractual penalty will produce effects that the government would prefer to prevent.

The contribution we expect to give to the field of knowledge involving the application of the real options approach to construction projects and BOT projects is manifold.

To the best of our knowledge, in Chapter II we propose the first model contributing to the optimal mark-up bid debate applying the real options approach. By identifying and evaluating the option to sign the contract and invest in performing the project, and considering that the option can only be exercised by the selected bidder, we suggest a numerical solution, consisting of a maximization problem, whose outcome is the optimal price contractors should include in the bid proposals. We believe that, by identifying and evaluating this option and, hence, recognizing the existence of flexibility to the selected bidder as of whether to sign the contract and, consequently, to invest in performing the project, the model addresses the optimal mark-up bid debate from an innovative perspective.

In Chapter III, our contribution is mainly focused in how we approach a specific type of uncertainty surrounding construction projects and the method applied to assess the expected impact that this specific type of uncertainty produces on the project value. We designate this type of uncertainty as "volume uncertainty". Our contribution consists in evaluating the impact of volume uncertainty on the project value, during the bid preparation stage, and incorporate this impact on the decision making process regarding the definition of the price to include in the bid proposal. A contribution is also given concerning the method used to assess that same impact by including in the suggested model a discrete-time stochastic variable, designated as "additional value". The model's outcome is a decision rule managers should use regarding the amount of incremental investment in human capital and technology that may be undertaken with the purpose of determining the true value for the expected additional profit.

In Chapter IV, we propose the first model, addressing a research topic in the context of a BOT project, based on a two-factor uncertainty approach in continuous-time. To the best of our knowledge, the only research piece that assumes the two key-value drivers of a BOT project as stochastic variables is the paper by Ho and Liu (2002). However, these authors address a completely different research question and the model is based on a discrete-time framework.

The work included in Chapter IV also provides a contribution to the current body of research by proposing an innovative contractual framework, in the context of BOT projects, whereby the public entity grants leeway to the selected bidder regarding the timing for project implementation, although a legal penalty may be enforced in the event the selected bider does not implement the project immediately. The model's outcome is the optimal value for this contractual penalty. Another contribution to this field of knowledge is suggested by incorporating two factors in the model, which we believe have not been considered in previous research pieces where the real options approach is used to address topics concerning BOT projects: the existence of social costs and the generally accepted argument that the private entity is more efficient than the government in executing the project facility.

Chapter 2

Reaching an Optimal Mark-Up Bid through the Valuation of the Option to Sign the Contract by the Selected Bidder

2.1 Introduction

In this Chapter, we aim to reach an optimal profit margin in the context of a bidding competition process applying the real options approach. The model herein suggested is a theoretical model whose purpose is to optimize the contractor's price through the valuation of the option to sign the contract and invest in performing the project. When a contractor presents a bid proposal to the client, and assuming that the probability of winning the bid is greater than zero, the option to sign the contract and, subsequently, to invest in executing the project does have value, as clearly established in the option pricing theory. The motivation behind the present research is also supported by the presence of uncertainty since the estimated costs of performing the project - the construction costs - will most likely vary from the moment the bidder computes them and establishes the price to include in her or his bid proposal based on such estimate, closes the proposal, delivers the proposal to the client, and the moment the option is exercised or not, *i.e.*, the moment the selected bidder is invited by the client to sign the contract and decides to sign it or declines the invitation. In fact, and even though the proposed bid price remains unchanged during this period, the uncertainty in construction costs will most likely lead to changes in the project's expected profit margin until the contract is eventually signed and the parties legally bounded.[1]

As far as the present research is concerned, contractors are firms operating in the construction industry whose business consists in executing a set of tasks previously defined by the client. The amount of tasks to be performed constitute a project, job or work. A significant amount of projects in the construction industry are awarded through what is known as "tender" or "bidding" processes (Christodoulou (2010); Drew et al. (2001)), being this the most popular form of price determination (Liu and Ling (2005); Li and Love (1999)). A bidding process consists of a number of contractors competing to perform a particular project by submitting a sealed proposal until a certain date previously defined by the client. The usual format of a bidding process is based on the rule that - all other things being equal - the contract will be granted to the competitor that submitted the lowest bid (Cheung et al. (2008); Chapman et al. (2000)), *i.e.*, the lowest price. Bearing this in mind, it is easy to conclude that the client's decision is very straightforward but the contractor's decision on what price to bid is more difficult to reach, being probably one of the most difficult decisions construction managers have to face during the bid preparation process (Li and Love (1999)).

The construction industry is known for featuring strong levels of price competitiveness (Chao and Liu (2007); Mochtar and Arditi (2001); Ngai et al. (2002)) and the competitive pressures are probably more intense than in any other industry (Drew and Skitmore (1997); Skitmore (2002)). This fact often leads contractors to lower their profit margins in order to produce a more competitive bid. Thus, it is not rare to see the winning bid include a near zero-profit margin (Chao and Liu (2007)). Moreover, under-pricing in the context of competitive bidding is a common phenomenon, namely explained by the need for work and penetration strategies (Drew and Skitmore (1997); Fayek (1998); Yiu and Tam (2006)), even tough bidding below-cost does not necessarily guarantee a successful result to the bidder (Tenah and Coulter (1999)).[2]

Contractors recognize the existence of this fierce price competition and realize that bidding low increases the chance of being selected to perform the project but they are also aware of the opposite: if the price included in their proposal is higher, the probability of winning the bid will definitively be lower. This inverse relationship between the level of the profit margin (commonly known in the construction management literature as the "mark-up bid")

[1]The risk that derives from the existence of uncertainty concerning the expected construction costs cannot be hedged since the bid participants do not know how the bidding process will end.

[2]We believe that the expression "under-pricing" is used to reflect the inclusion of profit margins in the bid price lower than the ones contractors would include if the price competition was not perceived as being particularly intense. Hence, under-pricing is not necessarily the same as bidding below-cost.

and the probability of winning the contract is an accepted fact both in the construction industry and within the research community (see, for example, Christodoulou (2010); Kim and Reinschmidt (2006); Tenah and Coulter (1999); Wallwork (1999)).

Competitive bidding has been a subject of research since the important papers of Friedman (1956) and Gates (1967) set the standards for future discussion. Both models proposed a probabilistic approach to determine the most appropriate mark-up bid and were supported by the existence of a relationship between the mark-up value and the probability of winning the bid. For that purpose, the two authors assumed the existence of previous bidding data - leading to the definition of the bidding patterns of potential competitors. Gates (1967) had the merit to extend the model built by Friedman (1956) and turned it into a strategic model, with general applicability, setting the foundations for what is now commonly known as "Tendering Theory" (Runeson and Skitmore, 1999). Later attempts to establish a relationship between the level of the profit margin and the probability of winning the bid were based on previous bidding data – in line with the mentioned pioneer models. Carr (1982) proposed a model similar to Friedman's but differing in the partitioning of the underlying variables: Friedman (1956) used a single independent variable, a composite "bid-to-cost" ratio, whereas Carr (1982) crafted his model around two distributions: one that standardizes the estimated cost of the analyzing bidder to that of all competitor bids, and another that standardizes the bids of an individual competitor against that of the analyzing bidder's estimated costs. More recently, Skitmore and Pemberton (1994) presented a multivariate approach, assuming that an individual bidder is not restricted to data for bids in which he or she has participated, as in the case of Friedman (1956) and Gates (1967) models. Instead, the bidder is able to incorporate data for all bidding competitions in which competitors and potential competitors have participated, regardless of the individual bidder's participation. This methodology had the merit of increasing the amount of data available for estimating the model's parameters. An optimal mark-up value is then reached against known competitors, as well as other types of strategic mark-ups.

Past research seems to suggest that it would be difficult to establish a link - with general applicability - between the mark-up level and the probability of winning the bid. Contractors may recur to previous bidding data and assume that bidders are likely to bid as they have done in the past, in order to shape the relationship that best describes their specific situation. However, as Fayek (1998) stated, past bidding information is not always available. To clearly understand what researchers mean by "past bidding information", we should distinguish between two different types of bidding data: (i) the one that is available to all contractors and comprises the estimates carried out by the client's engineers for the execution of each task or set of tasks, the price of each competitor for the execution of such task or set of tasks and the final bid price of each competitor, *i.e.*, the price of executing all the tasks defined by the client; (ii) the real empirical data each contractor (eventually) compiles regarding the results reached in past bidding competitions when a specific mark-up bid was included in the bid price. The first type of data is publicly available and allows researchers to reach, through the application of several models and methodologies, what are commonly known as "theoretical probabilities". These probabilities are determined based on data which is not real empirical data. Real empirical data is private information that contractors seldom share, meaning that this information is rarely observable and is, therefore, considered private knowledge of each contractor.[3] However, we recognize that assuming bidders are likely to bid as they have done in the past becomes inevitable, regardless of the type of data in question. In fact, utilizing past bidding information is only useful if one assumes that other bidders will decide in the future in the same way they have decided in the past. Still, and even though we agree that this assumption (which has been adopted since the pioneer works of Friedman (1956) and Gates (1967)) may be considered somewhat restrictive, we sympathize with Crowley (2000) when this researcher argued that bid models do not predict the future, but simply organize past bidding information in a way that is meaningful to current bid decisions.[4]

Most of the more recent contributions to the optimal mark-up bid debate have been concerned with the selection of factors construction managers should take into account when deciding what price to bid (Christodoulou, 2010). Research by authors such as Drew and Skitmore (1992), Shash (1993) and Drew et al. (2001) observed that different bidders apply different mark-up policies, which may be variable or fixed. These authors list a long set of factors aiming to explain the *rationale* behind mark-up bidding decision making: (1) amount of work in hands; (2) number and size of bids in hands; (3) availability of staff, including architects and other supervising officers; (4) profitability; (5) contract conditions; (6) site conditions; (7) construction methods and programme; (8) market conditions and (9) identity of other bidders, to name the ones they considered to be the most important. In general terms, factors are grouped in different categories and we sympathize with the five categories

[3]Chapman et al. (2000) stated that many construction managers argue that collecting the necessary information to apply quantitative models is too difficult, too expensive or even impossible - hence acknowledging the fact that not all contractors compile data from previous bidding competitions.

[4]We believe that past bidding information is, in fact, the best tool construction managers may use to create a perception as of how bidders will tend to act in the future. However, since each project has characteristics that distinguishes it from all previous projects, we argue that construction managers should also consider the specific features of the current bidding process when defining the mark-up bid.

defined by Dulaimi and Shan (2002): (1) project characteristics; (2) project documentation; (3) contractor characteristics; (4) bidding situation and (5) economic environment. Following this line of thought, innovative research on the subject has been embracing more sophisticated methodologies. The paper by Li and Love (1999) manages to combine rule-based expert systems with Artificial Neural Networks (ANN) in the context of mark-up bid estimation, following previous research conducted by Li (1996), Moselhi et al. (1991), amongst others. In fact, the most recent and innovative models use ANN (as in Christodoulou (2010) and Liu and Ling (2005)) or Goal Programming Technique (Tan et al. (2009)), where those determinants (or attributes) provide the ground where models are built upon, thus recognizing the crucial importance of possessing a strong knowledge of the factors influencing the contractors bid mark-up decision for the purpose of establishing the optimal mark-up value (Dulaimi and Shan (2002)).

Nevertheless, several studies suggest that decisions regarding the definition of the mark-up bid are mainly supported using subjective judgment, gut feeling and heuristics (Hartono and Yap, 2011), hence acknowledging the fact that, at least managers have a perception in real-world situations as of how a specific mark-up level will affect the probability of winning the current competition. In fact, we can not state that all construction managers support their mark-up bid decisions using some kind of mathematical expression linking the profit margin and the probability of winning the contract, but they are aware that higher mark-up values will lead to lower chances of winning the project and do have a perception as of how their decision regarding the definition of the mark-up bid will affect the probability of being selected to perform the work. Bearing this in mind, we decided to propose a mathematical expression linking the mark-up level with the probability of winning the bid that (i) respects the generally accepted inverse relationship between these two variables; (ii) allows for flexibility and, thus, may be adapted to accommodate the unique circumstances that surround a particular bidding process.[5] Moreover, the mathematical relationship that we suggest is very similar to the one that results from the application of the Gates (1967) model included in Skitmore et al. (2007) and, with a specific calibration, is almost graphically equal to the one these researchers

[5]Chapman et al. (2000) stated that, even though the probability of winning the bid may be difficult to determine, a relationship between the price and the probability of winning are implicit in any bidding process. These authors also argue that, usually, the persons involved in the pricing decision have their own implicit version of such relationship, which drives their decision making process. Chapman et al. (2000) stressed that making these implicit perceptions explicit is an important part of arriving at an appropriate final bidding price. This line of thought reinforces our argument that managers - at least - do have a perception as of how a specific price will affect the probability of getting the contract and also underlines the importance of making explicit such implicit perception.

have reached.[6] These authors used publicly available empirical data, which is included in a previous study carried out by Schaffer and Micheau (1971).[7] In their research piece, Skitmore et al. (2007) applied three different methodologies: the Gates (1967) model, the exponential model and the Weibull model.[8] The mathematical relationship we suggest may be calibrated to match the graphic representation of the results reached by using the Gates (1967) model and, still, is sufficiently flexible to be adapted in order to explicitly shape the perception construction managers have regarding the effect of the mark-up level in the probability of winning the current contract.

Even though some work has been developed consisting in the application of the real options approach to the construction management field (Espinoza (2011); Tseng et al. (2009); Yiu and Tam (2006); Mattar and Cheah (2006); Ng and Bjornsson (2004); Ng and Chin (2004)) and, more specifically, aiming to evaluate a set of real options in the context of large-scale investments (Pimentel et al. (2012); Couto et al. (2012)), there seems to be a lack of research contributing to the optimal mark-up debate using this methodology, motivating us to build up a model embracing the real options approach and aiming to reach the optimal mark-up bid. This will be achieved by evaluating the option to sign the contract and invest in performing the project and weighting the value of this option by the probability of winning the bid, since the option can only be exercised by selected bidder. According to our model, construction managers should establish a price which corresponds to the highest value of the option to sign the contract, weighted by the probability of winning the bid. In financial terms and under the real options approach, this is the right perspective to follow: to the highest value of the option to invest - weighted by the probability of winning the contract - will correspond a certain value for the profit margin, this being the optimal mark-up bid.

The remainder of this Chapter unfolds as follows. In Section 2.2, each of the model's components is described and the model's numerical solution is proposed. In Section 2.3, a numerical example is presented and a sensitivity analysis is performed to the option volatility level, its 'time to expiration' and to the amount of the expected construction costs. We also assess the impact of variations in each of the calibration parameters included in the proposed math-

[6] We propose an equation linking the mark-up level with the probability of winning the contract which comprises two parameters that should be calibrated with the purpose of accommodating contractor's past bidding data and their perception of how the mark-up decision affects the probability of winning the contract, considering the specific features of the current bidding process.

[7] The empirical data used comprised the estimates done by the client's engineers in 50 bidding contracts and the prices presented by each of the bid participants.

[8] These authors stressed that, for the Gates (1967) model to be considered valid in the context of their work, they had to assume that bids can be described using the proportional hazard family of statistical distributions. Please refer to Skitmore et al. (2007) for further details.

ematical relationship between the mark-up bid and the probability of winning the contract on the option value and on the optimal price. In Section 2.4, we consider the existence of penalty costs if the selected bidder decides to decline the invitation to sign the contract and, consequently, does not perform the project. We adapt the model accordingly and present the new results based on the inputs of the numerical example presented in Section 2.3. Finally, in Section 2.5, conclusions and remarks are given.

2.2 The Model

2.2.1 Introduction

Our model proposes a different approach regarding how the mark-up bid decision should be made, recognizing the real options approach as an effective methodology in addressing the optimal mark-up bid debate since the model herein presented (i) features uncertainty concerning the behavior of the construction costs, from the moment the bid price is established and the moment the client invites the selected bidder to sign the contract;[9] (ii) considers flexibility regarding the decision to sign the contract and invest in performing the project by the selected bidder and (iii) recognizes that the investment expenditures are, at least, partially irreversible as construction costs are project-specific. The three characteristics that the literature identifies as being essential for applying the real options approach to evaluate investment decisions are, thus, present in the model we will describe.

2.2.2 Assumptions

We will assume that (i) each bidder decides what price to include in his or her proposal in isolation; (ii) each bidder prepares his or her proposal simultaneously with the other competitors; (iii) each bidder presents a single-sealed proposal to the client; (iv) each bidder has access to the available information concerning the project in hands and all documentation to support the cost estimation and the final bid decision, in line with all other potential bidders; (v) the "bid package" also contains information about the date the bid results will be available to all participants; (vi) it is possible to establish an inverse relationship between the mark-up

[9]Construction costs continue to behave stochastically after the selected bidder is invited to the sign the contract. However, in the context of the real option we have identified - which expires in the moment the selected bidder decides whether to sign the contract or not - such fact is not relevant.

value and the probability of winning the bid; (vii) the selected bidder will only decide if he or she is going to invest in executing the project at the moment the contract has to be signed and not before that date.

Our model is thus based on the existence of a single-sealed bidding competition where interaction or contact of any kind with other bid participants is not considered. We will further assume that bidders have no information about the number of competitors until the bid results are publicly available. We will first assume the absence of penalty costs in the case the selected bidder decides not to sign the contract. Later on, in Section 2.4, we will consider the presence of these costs, which may be of two different types: (i) financial costs, legally enforced by the client and/or (ii) reputational costs, *i.e.,* costs that may be borne by the contractor in future bidding competitions as a result of declining the invitation to sign the present contract.

2.2.3 Model Description

The model aims to determine the optimal mark-up bid to be included in the contractor's bidding price and depends upon two different components: (i) the value of the option to sign the contract and invest in performing the project, which will be modeled as a contingent claim, adapting the exchange option model proposed by Margrabe (1978); (ii) the probability of winning the bid, since this option is only available to the selected bidder. Therefore, the value of the option to sign the contract has to be weighted by the probability of winning the contract.

As the option pricing theory establishes, there is a positive relationship between the price included in the bid proposal and the value of the option to sign the contract and invest in executing the project. In fact, the option value increases as the "underlying asset", *i.e.*, the bid price increases. However, the higher the bid price the lower will be the probability of winning the contract, as we previously mentioned. Thus, variations in the bid price will produce opposite effects in the two components. Consequently, the optimal bid price will be the solution of a maximization problem. We now proceed to present the two components separately.

2.2.3.1 The Value of the Option to Sign the Contract and Perform the Project

The Margrabe (1978) exchange option model builds on the Black and Scholes (1973) formula, used to evaluate a typical european *call* option and considers the existence of only one stochastic variable: the price of the "underlying asset", whereas the Margrabe (1978) model incorporates two "underlying assets", being the model's outcome the value of an european *call* option to exchange one asset for another. Let P denote the price included in the bid proposal and K the expected amount for the construction costs computed during the bid preparation stage. We adapt the Margrabe (1978) exchange option model to accommodate the fact that only the exercise price is uncertain, *i.e.*, the construction costs, K and also accounting for the fact that $dP = 0$, which means that the present value of P must be determined[10]. K follows a stochastic process known as geometric Brownian motion, given by the following equation:

$$dK = \alpha K dt + \sigma K dz \tag{2.1}$$

where α is the drift parameter, dt is the time interval, σ is the standard deviation (volatility parameter) and dz is the increment of a standard Wiener process. The Margrabe (1978) formula (F) becomes:

$$F(P,K) = Pe^{-r(T-t)}N(d_1) - KN(d_2) \tag{2.2}$$

being (d_1) and (d_2):

$$d_1 = \frac{ln(P/K) - (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{2.3}$$

$$d_2 = d_1 - (\sigma\sqrt{T-t}) \tag{2.4}$$

$N(d_1)$ and $N(d_2)$ are the probability density functions for the values resulting from expressions (d_1) and (d_2), respectively. σ^2 is the variance which, in our model, equals σ_K^2 [11], r is the risk-free interest rate and $T-t$ is the time between the moment the bid price is established and the moment the contract has to be signed.

[10]Taking into account the fact that the exercise price, *i.e.*, the construction costs behave stochastically, we could have evaluated this option as if it was a *put* option.

[11]$\sigma^2 = \sigma_P^2 - 2\sigma_P\sigma_K\rho_{PK} + \sigma_K^2$, where ρ_{PK} is the correlation coefficient between the price, P and the construction costs, K. Since P remains unchanged, then σ_P^2 equals zero and so does $2\sigma_P\sigma_K\rho_{PK}$.

2.2.3.2 The Probability of Winning the Bid

Based on our previous considerations, we propose an inverse relationship linking the mark-up ratio, (P/K) and the probability of winning the bid, $W(P,K)$, which is given by the following equation:

$$W(P,K) \quad = \quad e^{-b(P/K)^n} \qquad (2.5)$$

where 'n' and 'b' are parameters that should be used to calibrate the expression linking the mark-up ratio and the probability of winning the contract in order to best reflect each contractor specific circumstances, as we previously argued. We will show how each of these parameters affect the graphical representation of equation (2.5).

Parameter 'n'

Parameter 'n' is responsible for shaping the graphical configuration of equation (2.5) in terms of its concavity and convexity. Assuming parameter 'b' equals $ln(1/0.5)$, Figure 2.1 illustrates the impact caused on the configuration of equation (2.5) by five different values for parameter 'n'.

Figure 2.1: representation of the equation linking the mark-up ratio with the probability of winning the bid, considering five different values for parameter 'n'

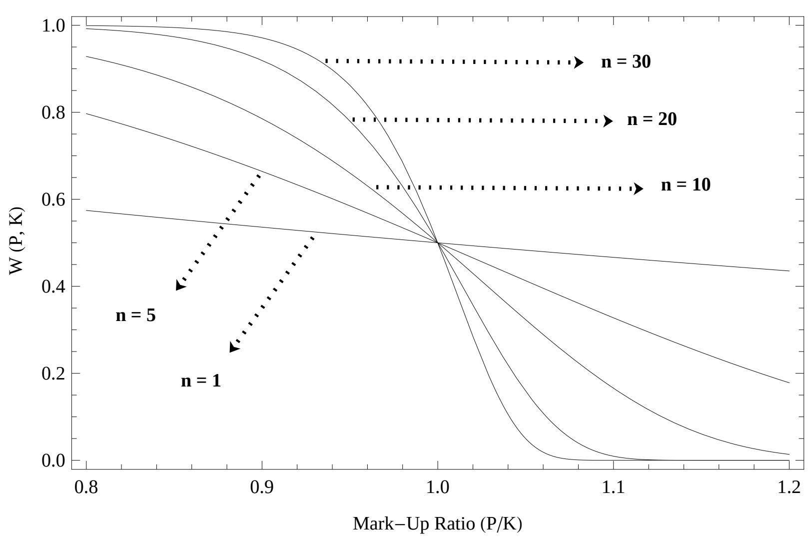
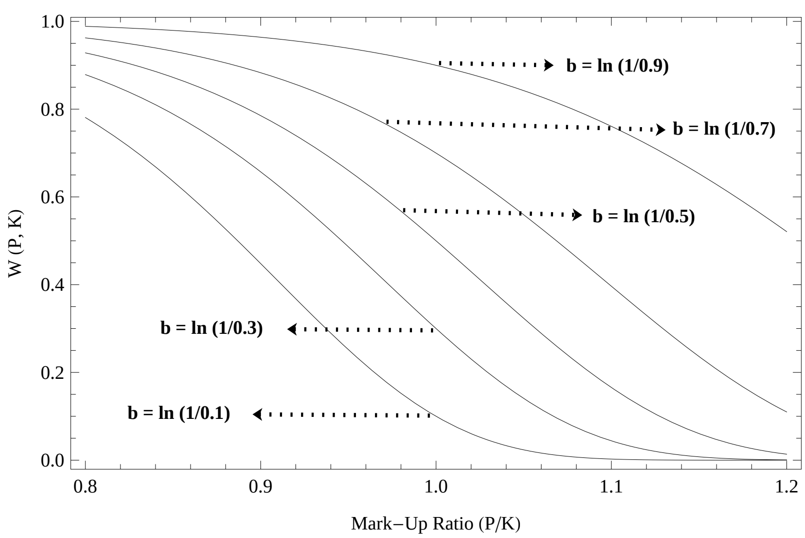


Figure 2.1 shows that the curve becomes more pronounced as parameter 'n' assumes higher values. In its concave region, higher mark-up ratios cause increasing variations in the probability of winning the contract whereas, in the convex region, higher mark-up ratios lead to decreasing variations in the probability of winning the bid. This effect assumes more importance as the curve stretches out in response to greater values assumed by parameter 'n'. For lower values of 'n', the relationship between the mark-up ratio and the probability of winning the bid becomes almost linear. In fact, the function shrunks towards the center, gradually showing both weaker concavity and convexity and, thus, traducing lower sensitivity to variations in the mark-up ratio. In real-world situations, managers should calibrate this parameter and establish the existence and the pace at which this effect takes place.

Parameter '*b*'

Assuming parameter 'n' equals 10, Figure 2.2 shows the impact on the configuration of equation (2.5), resulting from considering five different values for parameter 'b'.

Figure 2.2: representation of the equation linking the mark-up ratio with the probability of winning the bid, considering five different values for parameter 'b'



This parameter enables contractors to calibrate the functional relationship between the mark-up ratio and and the probability of winning the contract, given by equation (2.5), by setting the probability of winning the bid when the price includes a zero-profit margin (*i.e.*, when the mark-up ratio equals 1). Figure 2.2 shows that, the greater the probability of winning the contract considering a zero-profit margin (the configuration is presented for 10%, 30%, 50%, 70% and 90% probability of winning the bid with a zero-profit margin), the more shifted up and to the right the curve becomes. Thus, the greater the probability of winning the contract with a mark-up bid equal to 1, the less prominent the convex region is, thus reflecting that variations in the mark-up ratio for values situated in this area will cause smaller decreasing impacts on the probability of winning the bid; on the other hand, the concave region becomes more prominent and variations in the mark-up ratio located in this area will lead to greater increasing variations in the probability of winning the contract.[12]

2.2.3.3 The Optimal Price

The optimal price will be the one that maximizes the value of the option to sign the contract and invest in performing the project weighted by the probability of winning the bid. Thus, the model's outcome is the solution for the following maximization problem:

$$V(P,K) = \max_{P} \left\{ [Pe^{-r(T-t)}N(d_1) - KN(d_2)][e^{-b(P/K)^n}] \right\} \tag{2.6}$$

Therefore, to the highest value of the option weighted by the probability of winning the bid will correspond a specific price, P and, therefore, a specific mark-up value, $M = P - K$ and the corresponding mark-up ratio, P/K. This will be the optimal price, P^*, the optimal margin, $M^* = P^* - K$ and the optimal mark-up ratio P^*/K, as we illustrate in the following numerical example.

2.3 Numerical Example

2.3.1 The Base Case

Table 2.1 includes information about the inputs used in our numerical example.

[12] If we calibrate equation (2.5) with parameter 'b' approximately equal to ln(1/0.225) and parameter 'n' equal to 12, and also consider that the mark-up ratio ranges from 0.75 to 1.15, the graphical representation of equation (2.5) almost matches the one Skitmore et al. (2007) reached by applying the Gates (1967) model.

Table 2.1: inputs: description and values

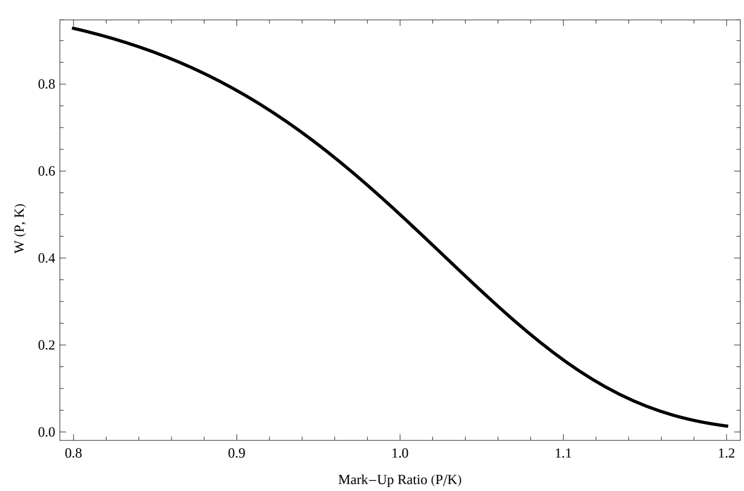
input	description	value
K	construction costs	€ 50,000,000
σ	standard deviation	25%
r	risk-free interest rate	1%
$T-t$	time from the moment the price is established until the contract is awarded	0.5 (years)
n	parameter for calibrating the relationship between P/K and W	10
b	parameter for calibrating the relationship between P/K and W	ln(1/0.5)

Considering the values included in Table 2.1, the relationship between the mark-up ratio and the probability of winning the bid will be given by the following equation:

$$W(P,K) = e^{-ln(1/0.5)(P/K)^{10}} \tag{2.7}$$

Hence, we are considering that there is a 50% probability of winning the bid if the contractor establishes a zero-profit margin, *i.e.*, $P/K = 1$, and also that parameter 'n'$= 10$. Figure 2.3 shows the configuration that results from this specific calibration, considering that the mark-up ratio ranges from 0.8 to 1.2.

Figure 2.3: graphical representation of the equation linking the mark-up ratio with the probability of winning the bid, considering that parameter 'b' equals $ln(1/0.5)$ and parameter 'n' equals 10



This inverted S-shaped curve respects the generally accepted inverse relationship between the mark-up ratio and the probability of winning the bid and typically comprises two regions: a concave region and a convex region. In its concave region, variations in the mark-up ratio lead to increasing variations in the probability of winning the contract whereas, in the convex region, variations in the mark-up ratio cause decreasing variations in the probability of winning the bid. Table 2.2 includes a set of representative values for both variables, applying equation (2.7):

Table 2.2: representative values for the mark-up ratio and the probability of winning the bid (for: b = ln (1/0.5); n = 10)

P/K	0.800	0.850	0.900	0.950	1.000	1.050	1.100	1.150	1.200
W	92.8%	87.2%	78.5%	66.0%	50.0%	32.3%	16.6%	0.61%	0.14%

Table 2.3 includes the results reached considering a set of different prices, P, the corresponding mark-up values, M and mark-up ratios, P/K.

Table 2.3: different results for the option value, considering different price levels (for: K = € 50,000,000; b = ln (1/0.5); n = 10; σ = 0.25; r =0.01; $T-t$ = 0.5 years)

P (€)	P/K	$M(P,K)$ (€)	$F(P,K)$ (€)	$W(P,K)$	$V(P,K)$ (€)
40,000,000	0.8000	-10,000,000	365,301	92.83%	339,109
45,000,000	0.9000	-5,000,000	1,353,040	78.53%	1,062,542
50,000,000	1.0000	0,000,000	3,389,530	50.00%	1,694,765
50,420,898	**1.0084**	**420,898**	**3,612,312**	**47.05%**	**1,700,224**
55,000,000	1.1000	5,000,000	6,520,306	16.57%	1,080,415
60,000,000	1.2000	10,000,000	10,498,738	1.37%	143,833

The results included in Table 2.3 demonstrate that, the higher the value of the "underlying asset", the higher the value of $F(P,K)$ (the value of the option to sign the contract increases in response to the presence of higher bid prices) and the lower the value of $W(P,K)$, since this probability decreases as the bid price (or mark-up bid) assumes greater values. This combined effect - reflecting opposite responses to variations in the bid price - makes $V(P,K)$ increase until its maximum value is reached: € $1,700,224$. To this maximum value of the option to invest weighted by the probability of winning the bid, $V(P,K)$ corresponds the optimal mark-up value, M^*= € $420,898$ and the optimal mark-up ratio, $P/K^* = 1.0084$. Thus, the optimal price is P^*= € $50,420,898$.

Figure 2.4 shows the relationship between the price, P and the option value, $V(P,K)$, considering the inputs included in Table 2.1.

Figure 2.4: relationship between the price and the option value

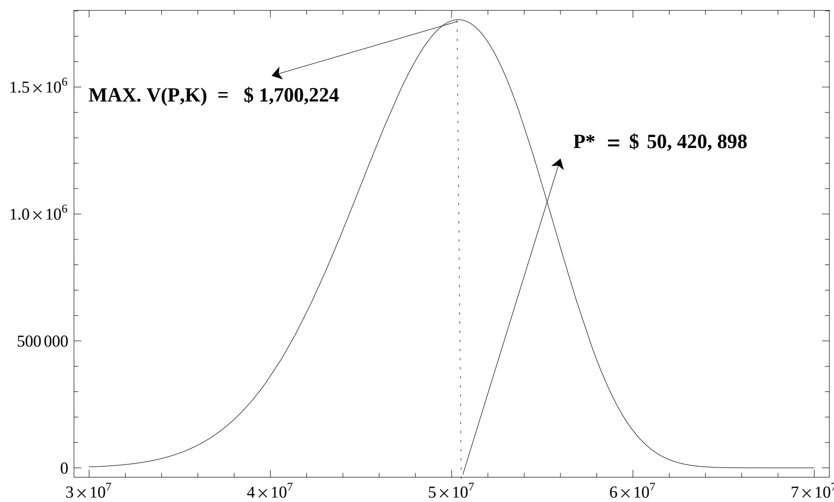


Figure 2.4 illustrates the maximization problem given by equation (2.6). The function increases until it reaches the maximum value for the option to invest, *i.e.*, $V(P,K) =$ € 1,700,224, to which corresponds the optimal price, $P^* =$€ 50,420,898. From this price onwards, the option value, $V(P,K)$ decreases and tends to zero, as the price tends to infinity.

2.3.2 Sensitivity Analysis

We first perform a sensitivity analysis to two of the parameters that influence the value of the option before being weighted, $F(P,K)$ - the volatility parameter and the 'time to expiration' parameter - with the purpose of assessing the impact of three different values for each of them on the option value, $V(P,K)$ and on the optimal price, P^*. We also perform a sensitivity analysis to the expected amount of direct costs of executing the project, in order to verify if a scale-effect is present in the model. Finally, we present the results of the impact caused by considering different values for each of the parameters included in equation (2.5), *i.e.*, parameter 'b' and parameter 'n', on the option value, $V(P,K)$ and on the optimal price, P^*.

2.3.2.1 Volatility

Table 2.4 presents the impact of different volatility levels, σ on the option value, $V(P,K)$ and on the optimal price, P^*.

Table 2.4: sensitivity analysis: volatility parameter

(for: K = € 50,000,000; b = ln (1/0.5); n = 10; r = 0.01;$T-t$ = 0.5 years)

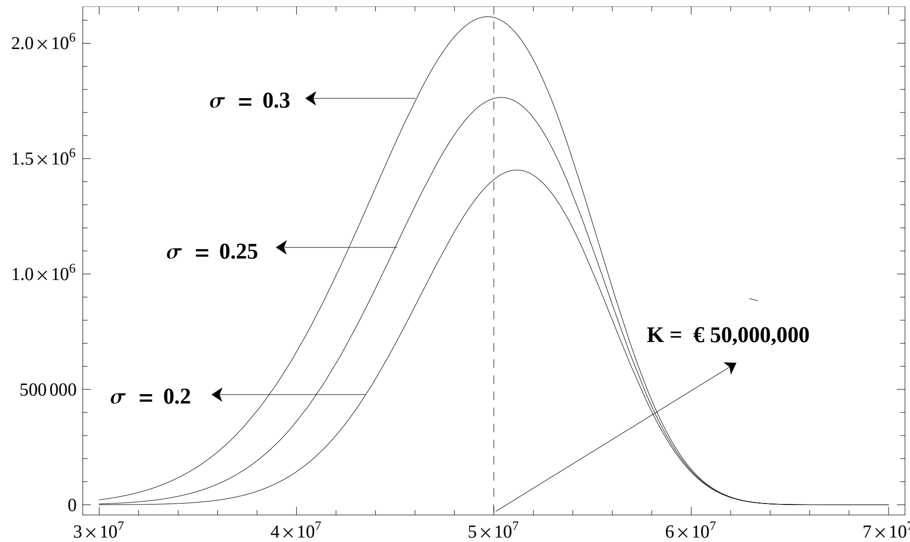
Volatility (σ)	**Option Value** $V(P,K)$ **(€)**	**Optimal Price** (P^*) **(€)**
0.15	1,123,521	52,145,144
0.175	1,248,923	51,672,800
0.2	1,388,233	51,226,901
0.225	1,539,224	50,809,755
0.25	1,700,224	50,420,898
0.275	1,869,923	50,058,683
0.2792	**1,899,209**	**50,000,000**
0.3	2,047,272	49,721,038
0.325	2,231,402	49,405,824
0.35	2,421,585	49,110,994

The results included in Table 2.4 clearly reflect that, the higher the volatility level the higher the maximum value of the option to invest, as the option pricing theory states. However, the optimal price - the one which corresponds to the highest value of the option - decreases since contractors will present a lower bid as a consequence of holding a more valuable option to invest.

We should stress that, according to these results, contractors will establish a price equal to the construction costs (*i.e.,* a mark-up bid equal to 1) if the volatility associated with the construction costs is 0.2792.

Figure 2.5 displays the different configurations that result from considering a set of three different levels of volatility: $\sigma = 0.2$; $\sigma = 0.25$ and $\sigma = 0.3$.

Figure 2.5: sensitivity analysis: volatility parameter



2.3.2.2 'Time to Expiration'

Table 2.5 presents the impact produced by considering three different values for the 'time to expiration' parameter $(T-t)$ on the option value, $V(P,K)$ and on the optimal price, P^*.

Table 2.5: sensitivity analysis: 'time to expiration' parameter

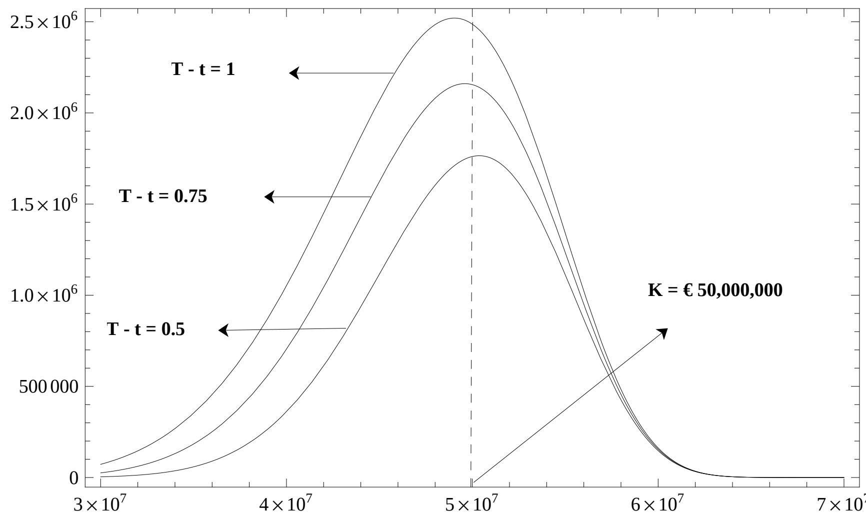
(for: K = € 50,000,000; b = ln (1/0.5); n = 10; σ = 0.25; r = 0.01)

Time to Expiration, $(T-t)$ (Years)	**Option Value, $V(P,K)$ (€)**	**Optimal Price, (P^*) (€)**
0.50	**1,700,224**	**50,420,898**
0.75	2,058,702	49,660,977
1.00	2,379,473	49,105,003

The results included in Table 2.5 are consistent with the ones we have reached in the last analysis: the longer the life of the option the greater the maximum value for the option to invest (as established by the option pricing theory) and the lower the optimal price, as we previously explained.

Figure 2.6 shows the different configurations that result from considering each of the values for the 'time to expiration parameter' included in Table 2.6.

Figure 2.6: sensitivity analysis: 'time to expiration' parameter



2.3.2.3 Construction Costs

Table 2.6 includes the results of the impact caused by a set of values for the construction costs, K on the option value, $V(P,K)$, on the optimal price, P^* and on the optimal mark-up ratio, P^*/K.

Table 2.6: sensitivity analysis: construction costs

(for: b = ln (1/0.5); n = 10; σ = 0.25; r = 0.01; $T-t$ = 0.5 years)

K (€)	$V(P,K)$ (€)	P^* (€)	(P^*/K)
12,500,000	425,096	12,605,224	1.0084
25,000,000	850,112	25,210,449	1.0084
50,000,000	**1,700,224**	**50,420,898**	**1.0084**
75,000,000	2,550,336	75,631,346	1.0084
100,000,000	3,400,447	100,841,795	1.0084
125,000,000	4,250,559	126,052,245	1.0084

The results included in Table 2.6 demonstrate that no scale-effect is present. Regardless of the level of construction costs, the optimal mark-up ratio remains unchanged, which means that the model's outcome responds linearly to variations in the project dimension.

2.3.2.4 The Impact of Variations in Parameter 'b' on the Option Value and on the Optimal Price

Table 2.7 exhibits the results produced by considering a set of different values for parameter 'b' on the option value after being weighted by the probability of winning the contract, $V(P^*, K)$, on the probability of winning the bid, $W(P^*, K)$ and, finally, on the optimal price, P^* and on the optimal mark-up ratio, P^*/K. We set parameter 'n' to 10.

Table 2.7: the impact of variations in parameter 'b' on the option value and on the optimal price

(for: K = € 50,000,000; n = 10; σ = 0.25; r = 0.01; $T-t$ = 0.5 years)

	$ln(1/0.3)$	$ln(1/0.4)$	$ln(1/0.5)$	$ln(1/0.6)$	$ln(1/0.7)$	$ln(1/0.8)$	$ln(1/0.9)$
$V(P^*, K)$ (€)	1,096,022	1,369,742	**1,700,224**	2,125,739	2,719,739	3,657,428	5,567,042
$W(P^*, K)$	43.27%	44.41%	**47.05%**	49.15%	51.60%	54.73%	59.51%
P^*(€)	48,219,507	49,297,530	**50,420,898**	51,675,187	53,187,382	55,224,015	58,643,159
P^*/K	0.9644	0.9860	**1.0084**	1.0335	1.0637	1.1045	1.1729

Not surprisingly, the results included in Table 2.7 demonstrate that the higher the probability of winning the contract with a zero-profit margin (which is exactly what parameter 'b' establishes) the higher the option value, $V(P^*, K)$ and the higher the optimal price, P^* and the optimal mark-up ratio, P^*/K. Therefore, positive variations in this parameter produce positive variations in the option value before being weighted by the probability of winning the bid (as a result of considering higher optimal bid prices), positive variations in the probability of winning the contract and, hence, positive variations in the optimal price and in the optimal mark-up ratio.

2.3.2.5 The Impact of Variations in Parameter 'n' on the Option Value and on the Optimal Price

Table 2.8 includes the results of considering several different values for parameter 'n' and the impact on the option value, $V(P^*, K)$, on the optimal price, P^* and on the optimal mark-up ratio, P^*/K. The results that illustrate the impact on the value of the option before being weighted by the probability of winning the contract, $F(P^*,K)$ and on the probability of winning the bid, $W(P^*,K)$ are also displayed.

Table 2.8: the impact of variations in parameter 'n' on the option value and on the optimal price

(for: K = € 50,000,000; $b = ln(1/0.5)$; σ = 0.25; r = 0.01; $T-t$ = 0.5 years)

	1	5	10	15	20	30
$F(P^*,K)$ (€)	71,775,662	6,991,910	**3,613,654**	2,957,695	2,750,190	2,652,675
$W(P^*,K)$	18.33%	30.63%	**47.05%**	58.61%	66.41%	75.90%
$V(P^*,K)$ (€)	13,156,479	2,141,622	**1,700,224**	1,733,505	1,826,401	2,013,380
P^*(€)	122,385,365	55,642,728	**50,420,898**	49,139,865	48,700,195	48,486,755
P^*/K	2.4477	1.1129	**1.0084**	0.9828	0.9740	0.9697

The results included in Table 2.8 are not so straightforward to interpret as in the previous case. The option value, $V(P^*,K)$ decreases until parameter 'n' attains a certain value and increases from then onwards.[13] In fact, for lower values of 'n', the impact produced by the optimal price, P^* on the value of the option before being weighted by the probability of winning the contract, $F(P^*,K)$ is stronger than the impact of opposite nature that the optimal price produces on the probability of winning the bid, $W(P^*,K)$. Hence, the two effects combined lead to a lower option value, $V(P^*,K)$.[14] For higher values of 'n', the opposite occurs: the impact that variations in this parameter produce on the probability of winning the bid, $W(P,K)$ is stronger than the impact produced on the value of the option before being weighted, $F(P^*,K)$, hence resulting in higher values for the option to invest, $V(P^*,K)$.

Thus, by confronting these results with the graphical representation of the impact caused by different values for parameter 'n' on the probability of winning the contract - the ones considered in Figure 2.1 - we conclude that including low values for this parameter in equation (2.5) ('n' $= 1$ and 'n' $= 5$, in Figure 2.1) lead to an almost linear relationship between the mark-up ratio and the probability of winning the bid. This means that the presence of low levels for 'n' result in a configuration to equation (2.5) where both concavity and convexity regions are less pronounced and changes in this parameter (say, from $n = 1$ to $n = 2$) lead

[13]The option value, $V(P^*,K)$ starts increasing for values of 'n' approximately greater than 11.

[14]Please note that $V(P^*,K) = F(P^*,K) * W(P^*,K)$.

to less strong positive variations in the probability of winning the bid, $W(P^*,K)$ than the stronger negative impact that results from considering lower optimal bid prices for the value of the option before being weighted by the probability of winning the contract, $F(P^*,K)$. On the other hand, considering higher values for 'n' leads equation (2.5) to show both greater concavity and convexity. Hence, when 'n' assumes higher values, changes in this parameter (say, from 'n' = 15 to 'n' = 30) lead to a higher impact on the probability of winning the bid, $W(P^*,K)$ than on the value of the option before being weighted, $F(P^*,K)$ since, as we have observed, variations in the optimal mark-up ratio situated in the curve's concave (convex) region will produce more (less) than proportional variations in the probability of winning the contract. This implies that, when we consider higher values for parameter 'n', the impact produced by variations in this parameter on the probability of winning the bid is stronger than the impact caused on the value of the option before being weighted, $F(P^*,K)$, hence resulting in greater levels for the option value, $V(P^*,K)$.

Thus, we conclude that the option value, $V(P^*,K)$ and the optimal price, P^* are highly sensitive to changes in parameter 'n' when it assumes very low levels, *i.e.*, when the relationship between the mark-up ratio and the probability of winning the bid, given by equation (2.5), becomes closer to a linear relationship. However, as 'n' assumes higher levels (15, 20, and 30, in our example), the impact of variations in this parameter is much less strong. In fact, the negative variations observed in the optimal price in response to higher values of 'n' are not substantial, as the results included in Table 2.8 clearly reflect.

2.4 Considering the Existence of Penalty Costs

In the previous Sections we have not considered the existence of penalty costs, which means that we have been assuming that the selected bidder will not bear any type of costs if he or she decides to decline the invitation to sign the contract. Yet, in some legal environments, the selected contractor may have to pay a legal compensation if the option to sign the contract is not exercised. According to Halpin and Senior (2011), in the United States contractors are free to withdraw their bids without incurring in any penalties if that happens prior to the ending of the bidding period. However, if a contractor decides to withdraw the bid after that moment - and assuming that he or she is the selected bidder - a penalty equal to the difference between the second best proposal and the selected bid is legally imposed, even if the contract has not yet been signed. According to Halpin and Senior (2011), "this may occur in the event that the selected bidder realizes that he or she has underbid the project and that pursuing the work will result in a financial loss" (p.44).[15] In these circumstances, the client may exercise the legal right of receiving the difference between the two bid prices.

For the sake of convenience but also because - from the contractor's perspective - the expected penalty costs can be seen as a percentage of the construction costs, we will assume that g, the amount of the penalty costs, is estimated as being a percentage of K, the construction costs.[16] Thus, the payoff (at maturity) of the option to sign the contract, under these new conditions, will be:

$$Max[P - K_T; -gK_T] \tag{2.8}$$

Expression (2.8) entails that the contractor will chose to pay the legal penalty, "gK_T" if this cost is smaller than the financial loss given by the difference between P and K_T, *i.e.*, the expected profit the project will generate, at the moment the contract needs to be signed, T. On the contrary, if this difference is smaller than the amount of the legal compensation, gK_T, than the contractor will prefer to sign the contract and execute the job. Considering these new conditions, we again adapted the Margrabe (1978) formula. Equation (2.9) below includes two important changes when compared with equation (2.2):

1) The exercise price is now equal to "$(1-g)K$". The explanation resides in the fact that, if the contract is signed and the project performed, the contractor will invest the amount "K". On the contrary, if the contractor declines the invitation to sign the contract, he or she incurs in a penalty cost equal to the amount "gK". Thus, seen in incremental terms, "$(1-g)K$" becomes the exercise price if the contractor exercises the option to perform the project in the presence of these costs.

2) $N(d_2)$ is the risk-neutral probability that the option will be exercised at maturity (Nielsen, 1992) in the original Black and Scholes (1973) formula or, in other words, the probability that the option will finish "in-the-money" in a risk-neutral world (Smith, 1976). This also holds the same meaning in the Margrabe (1978) model. Hence, $[1 - N(d_2)]$ expresses the probability that the option will not be exercised at the maturity since it will not finish "in-the-money". In these circumstances, the contractor will prefer to support the penalty costs, "gK" and not sign the contract. Let P_g denote the bid price in the presence of penalty costs. The adapted version of Margrabe (1978) formula, $F_g(P_g, K)$ becomes:

[15] We believe the authors use the word "underbid" to express the fact that the selected bidder may realize, in the day the contract has to be signed, that the expected construction costs are greater than the bid price.

[16] As we previously mentioned, penalty costs may also assume the nature of reputational costs.

$$F_g(P_g, K) = [P_g e^{-r(T-t)} N(d_1) - KN(d_2) - gK(1 - N(d_2))] \quad (2.9)$$

where:

$$d_1 = \frac{ln[P_g/(1-g)K)] - (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \quad (2.10)$$

and

$$d_2 = d1 - (\sigma\sqrt{T-t}) \quad (2.11)$$

$N(d_1)$ and $N(d_2)$ are the probability density functions for the values resulting from expressions (d_1) and (d_2), respectively. σ^2 is the variance[17] and $T-t$ is the time between the moment the bid price is established and the moment the contract has to be signed.

Equation (2.5) still holds. Hence, under these new conditions, the model's outcome is the solution for the following maximization problem:

$$V_g(P_g, K) = \max_{P_g} \left\{ [P_g e^{-r(T-t)} N(d_1) - KN(d_2) - gK(1 - N(d_2))][e^{-b(P_g/K)^n}] \right\} \quad (2.12)$$

Using the same inputs considered in Table 2.1, Table 2.9 includes the results for $F_g(P_g, K)$, $W(P_g, K)$ and $V_g(P_g, K)$ considering a set of different prices, P_g, the corresponding mark-up values, M_g and mark-up ratios, P_g/K , and assuming that the penalty costs, g equal 2% of the construction costs.

[17]As in the case where penalty costs were not considered, σ^2 equals σ_K^2.

Table 2.9: the impact of different prices on the option value and on the probability of winning the bid, in the presence of penalty costs

(for: K = € 50,000,000; b = ln (1/0.5); n = 10; σ = 0.25; r = 0.01; $T-t$ = 0.5 years; g = 0.02)

P_g (€)	P_g/K	$M_g(P_g,K)$ (€)	$F_g(P_g,K)$ (€)	$W(P_g,K)$	$V_g(P_g,K)$ (€)
40,000,000	0.800	-10,000,000	-329,395	92.83%	-305,777
45,000,000	0.900	-5,000,000	847,793	78.53%	665,772
50,000,000	1.000	0,000,000	3,112,681	50.00%	1,556,341
50,971,124	**1.019**	**971,124**	**3,937,479**	**43.16%**	**1,589,381**
55,000,000	1.100	5,000,000	6,439,134	16.57%	1,066,965
60,000,000	1.200	10,000,000	10,547,559	1.37%	144,502

The value of the option to sign the contract, $V_g(P,K)$ assumes negative values for low levels of P_g, in line with the interpretation of expression (2.8). The option value, $V_g(P_g,K)$ becomes positive for price levels above € $41,882,195$ and reaches its maximum value when $V_g(P_g,K) =$€ $1,589,381$, to which corresponds a price, $P_g =$ € $50,971,124$. This is the optimal price, P_g^* in the presence of penalty costs, a slightly higher value than when penalty costs were not considered. Figure 2.7 illustrates the relationship between the price, P_g and the option value, $V_g(P_g,K)$ for an estimated level of penalty costs, $g = 0.02$.

Figure 2.7: relationship between the price and the option value, considering the existence of penalty costs

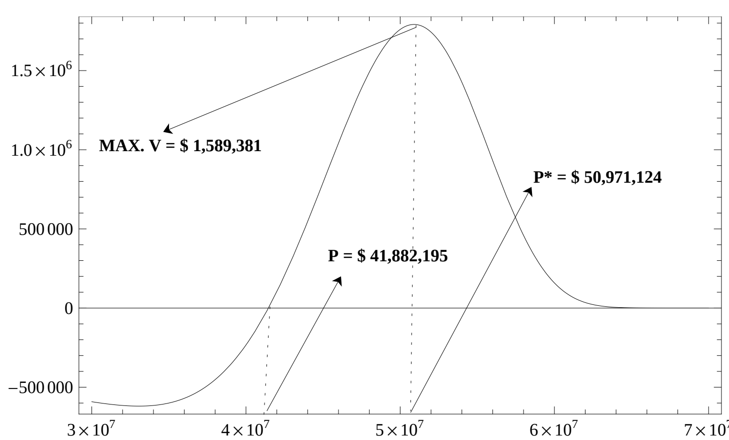
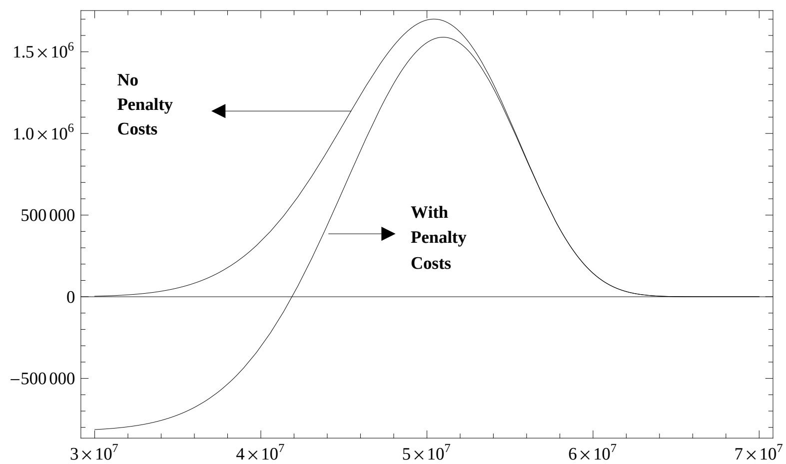


Figure 2.8 compares the relationship between the price, P and the value of the option to sign

the contract, $V(P,K)$ for both cases, *i.e.*, considering the base case where $g = 0$ and the case where penalty costs are considered and estimated to be $g = 0.02$.

Figure 2.8: relationship between the price and the option value, with and without the existence of penalty costs



In the ascending part of the two curves, the option value is higher in the absence of penalty costs for a specific price value. This difference becomes less important as the price increases, until the maximum value for the option is reached, in both scenarios. In their descending part, the curves feature very similar configurations, with the option value - in the presence of penalty costs - reaching a slightly lower value for a specific price level, until both curves converge when higher price levels are considered.

Table 2.10 includes the values for the optimal price, P_g^* that result from considering a set of three different levels of penalty costs, $g = 0.02$, $g = 0.04$ and $g = 0.06$.

Table 2.10: the impact of different levels of penalty costs on the option value and on the optimal price

(for: K = € 50,000,000; b = ln (1/0.5); n = 10; σ = 0.25; r = 0.01; $T-t$ = 0.5 years)

g (%K)	g (€)	$V_g(P_g,K)$ (€)	P_g^*(€)
2%	**100,000**	**1,589,381**	**50,971,124**
4%	200,000	1,512,369	51,368,786
6%	300,000	1,460,022	51,661,230

The results included in Table 2.10 reveal that the option value, $V_g(P_g,K)$ decreases as the penalty costs, g assume greater levels, reflecting the fact that the option to invest is less valuable when penalty costs are greater. The optimal price, P_g^* increases when greater levels of penalty costs are considered, which means that construction managers need to establish higher bid prices in order to compensate for the growing impact produced by this type of costs.

2.5 Conclusions and Remarks

The theoretical model herein presented aims to underline the importance of considering the existence of uncertainty and the presence of flexibility, at the bid preparation stage. We have identified and evaluated an option that is only available to the selected bidder - the option to sign the contract and invest in executing the project. The selected bidder has flexibility concerning the decision of whether to sign the contract and execute the project and, as clearly stated in the option pricing theory, flexibility does have value. The option to sign the contract and perform the project constitutes a real option and, when there are no penalty costs involved, the option should only be exercised if the construction costs, at the time the contract has to be signed, are lower than the price included in the bid proposal. However, when penalty costs are present, the selected bidder should only exercise the option if the difference between the bid price and the construction costs is greater than the penalty costs, in the day the contract has to be signed.

A numerical solution, which consists of a maximization problem, is proposed. This solution determines that - to the highest value of the option to execute the project weighted by the probability of winning the contract - corresponds the optimal price. According to the approach adopted, to this optimal price corresponds the optimal mark-up bid contractors should include in their proposals.

We performed a sensitivity analysis to three of the parameters that influence the value of the option and also assessed the impact of variations in each of the calibration parameters included in the suggested mathematical relationship between the price and the probability of winning the bid. The results revealed that the maximum value of the option is higher and the optimal bid price is smaller in response to positive variations in the volatility associated with the construction costs, as well as to positive variations in the 'time to expiration' parameter. The results also showed that the model's outcome responds linearly to variations in the amount of construction costs, which means that the optimal mark-up ratio remains unchanged for any dimension the investment may assume.

The sensitivity analysis performed to the calibration parameters revealed that the option value and the optimal price are highly sensitive to variations in each of them. The impact produced by considering higher values for parameter 'b' on the optimal price is considerable, and the higher the value assumed by this calibration parameter, the higher the option value and the higher the optimal bid price. However, when we examined the impact of variations in parameter, 'n', we concluded that the value of the option to sign the contract and perform the project decreases until a specific value assumed by this parameter, and increases from then onwards. When very low values for parameter 'n' are considered, the optimal price is highly sensitive to variations in this parameter and is much less sensitive to variations when 'n' assumes higher values. Therefore, construction managers should be aware of the high sensitivity that the optimal bid price exhibits in response to variations in parameter 'n' when it assumes very low levels. In fact, for low levels of 'n', any small variation will produce a great impact on the optimal bid price.

Finally, based on the inputs considered in the numerical example, we concluded that, when penalty costs are present, the optimal price is higher, corresponding to a lower maximum value of the option to invest in performing the project, when compared with the optimal price when penalty costs are absent. Furthermore, the optimal price increases in response to positive variations in the level of the penalty costs, as higher expected values of penalty costs lead construction managers to establish higher mark-up bids as a consequence of holding a less valuable option to invest. This increase in the price is the compensation construction managers demand for supporting the presence of greater levels of penalty costs.

Chapter 3

The Impact of Volume Uncertainty on the Project Value and on the Optimal Bid Price

3.1 Introduction

Uncertainty surrounding construction projects is a crucial element that should be adequately managed since it may have a considerable impact on construction company's overall performance. Construction companies or contractors are firms operating in the construction industry whose business resides in executing a set of tasks previously established by the client.[1] The amount of tasks to be performed constitutes a project, job or work. The vast majority of projects in the construction industry are assigned through what is known as "tender" or "bidding" processes (Christodoulou (2010); Drew et al. (2001)), this being the most popular form of price determination (Liu and Ling (2005); Li and Love (1999)). In a tender or bidding process, a certain number of contractors (bidders) compete to execute a project by submitting a single-sealed proposal until a specific date previously defined by the client. Potential bidders have access to a what is commonly known as the "bid package". This package contains a set of technical pieces (often also referred as "tender documents") which serve as the basis for establishing the price to include in the bid proposal. More specifically, the package includes plans and technical drawings, a proposal form, the "general conditions" covering

[1]For the purposes of this research, we exclude those situations where contractors execute their own projects, as it is often the case of construction companies operating in the real estate sector.

procedures which are common to all construction contracts and the "special conditions" containing the procedures to be used and that are unique to the project in question (Halpin and Senior (2011)), including information about the type of contract that will be enforced.

The usual format of a tender or bidding process is based on the rule that - all other things being equal - the contract will be awarded to the competitor that submitted the lowest price (Christodoulou (2010); Cheung et al. (2008); Chapman et al. (2000)) or, which is the same, the lowest bid. Thus, the bidder that proposed the lowest price will most likely be invited to sign the contract and, if the contract is actually signed, he or she will have to invest a substantial amount of money by incurring in the necessary direct costs to execute the project, *i.e.*, the construction costs.

Traditionally, construction management literature has been placing more emphasis on the negative effects of uncertainty, which means researchers seem to be more concerned with ways to deal with the risks involving the construction activities and how they may affect the value of the project through a negative impact on the construction costs. In fact, few authors have been addressing uncertainty as a source of opportunity, as it is the case of Ford et al. (2002), Ng and Bjornsson (2004) and Yiu and Tam (2006). Ford et al. (2002) argued that construction projects may include specific sources of uncertainty that affect project value, but not necessarily just by reducing it. This argument is supported by Ng and Bjornsson (2004) when they state that, even though uncertainty can lead to cost over-runs and delays, it can also produce positive return if properly managed. Following this line of thought, Yiu and Tam (2006) applied the real options approach to evaluate the intrinsic value of uncertainty and the managerial flexibility deriving from the options to defer and to switch modes of construction. Therefore, uncertainty can also be seen as a source of opportunities, rather than just an element that may cause undesirable effects during the construction stage - in clear opposition to the traditional view that "all uncertainty presumes loss" (Mak and Picken (2000)).

Ford et al. (2002) acknowledge the fact that many construction project conditions evolve over time and, thus, managerial choices for effective decision-making cannot be completely and accurately determined during the pre-project planning period. In fact, these authors observe that many aspects of construction projects are uncertain, such as input prices, the weather conditions, the length of some activities and the overall duration of the project, among others, meaning that the effects of some of these sources of uncertainty can only be recognized and properly managed as the project unfolds. This argument is also supported by Mattar and Cheah (2006) when they mention that contractors typically learn more about the value of the project as they invest over time and uncertainties are resolved.

Even though we recognize the fact that the possible consequences of some sources of uncertainty cannot be anticipated, we do believe that others can be predicted and accounted for during the bid preparation process. Moreover, we will try to demonstrate that it is possible to establish a support decision model that accommodates the expected impact of a specific type of uncertainty - the uncertainty associated with the amount of work to be executed during the project's life cycle - on the project value.

The model we propose in the present Chapter builds on this crucial aspect by focusing on a specific source of uncertainty which may lead to a greater project value by increasing the expected amount of work to be executed during the construction stage. This means we believe that this source of uncertainty is - at least - as decisive as the others in adding value to the project. Therefore, managers should recognize its importance by planning and strategically managing this element in a way that improves the project value and, as we will demonstrate, their competitiveness in a bidding competition context.

Despite the fact that project value can be substantially increased by reducing costs, we would like to reinforce the idea that project value can also be increased by raising more income. As we will see, more income means the income that is generated through actually executing, during the construction phase, a certain amount of tasks which were not included in the tender documents. We are thus concerned with the uncertainty that may lead to more project value by increasing the amount of work to be performed by contractors, "vis-a-vis" with the amount of work contractors are contractually bound to execute. We will refer to this type of value as "hidden-value".[2]

Hidden-value should be captured and quantified in the pre-project stage while the bid proposal is being prepared, by carefully analyzing the portions of the project where it may be concealed. Ford et al. (2002) observed that hidden-value is present in the most uncertain portions of the project, enabling us to sustain that skilled engineers and experienced managers - whose responsibility is to prepare the bid proposal - have a fairly good knowledge, based on their accumulated experience, of "where to look for". Chapman et al. (2000) stated that the bid preparation process begins with a preliminary assessment of the tender documents. We sympathize with this statement and argue that, in this preliminary assessment, it is possible to recognize and quantify hidden-value and, more specifically, to stipulate a high-estimate

[2] To the best of our knowledge, this designation was first adopted by Ford et al. (2002) and, in the context of their research, the definition encompasses other sources of hidden-value, rather than just those that may result in additional income.

and a low-estimate to this hidden-value and to attribute a probability of occurrence to each of the estimates just by undertaking a preliminary analysis of the tender documents.[3] However, the quantification of hidden-value with accuracy is, in most practical situations, a goal that can only be achieved by performing an exhaustive investigation of all the bid documents and performing studies that often require the use of specific advanced technologies, which many contractors do not possess. Therefore, many construction companies will have to invest in human capital and technology by recurring to external firms with the purpose of supplying managers with more accurate information concerning the project in hands, as Kululanga et al. (2001) stated.[4] In fact, these authors argue that an awareness of job factors, which may give rise for claiming extra-revenues due to extra-work to be executed is a skill that, generally, has to be specially acquired. Pinnell (1998) reinforces this argument when he mention that the individual (or the team) responsible for thoroughly analyzing the bid documents aiming to capture and quantify hidden-value during the bid preparation process may be a consultant expert or a team of consultant experts. Whether this incremental investment in hiring skilled consultants and contracting highly specialized firms aiming to supply contractors with more accurate information regarding the volume of work to be performed will be worthwhile constitutes the question we will address using the model proposed in Section 3.6.

Our model is thus based on the argument that uncertainty can add value to construction projects through the impact caused on the amount of work to be executed during the project's life cycle. This argument entails that contractors do not know, before the completion of the project (or, at least, before the job begins), how much volume of work will actually be executed. Hence, uncertainty is present concerning the volume of work, allowing us to designate this specific type of uncertainty, from the contractor's perspective, as "volume uncertainty". Volume uncertainty leads to uncertainty about the project's final value since the execution of additional work implies receiving extra income (or extra revenues) and incurring in extra costs. We will designate, from now on, the difference between these extra revenues and these extra costs as "additional value". Additional value is, therefore, the value that may be generated because there is, at least, a specific source of uncertainty surrounding construction projects that may actually cause such effect. We now proceed to discuss this subject with more detail.

[3] Our model will just consider a specific type of hidden-value: the one that may result in the creation of additional profit through the execution of more volume of work.

[4] Specialized firms are frequently hired with the purpose of providing the contractor with more accurate information concerning the soil and under-soil conditions he or she will encounter in the beginning of the construction stage. Not rarely, these studies indicate that the type of foundations defined by the client and included in the bid package is not safe or may need to be preceded by a deeper drilling work.

3.2 Recognizing and Quantifying Hidden-Value: The Concept of Additional Value

In many construction projects value is hidden in the most uncertain portions of the project, as we previously mentioned. After its detection and quantification, hidden-value becomes what we designate as additional value. To fully understand how hidden-value may be detected and properly quantified - and, hence, transformed into additional value - we must first know where hidden-value can be detected, which means we have to understand the nature of its sources.[5]

The construction management literature has been dedicating considerable attention to a subject commonly known as "Claims". A construction claim can be defined as "a request by a contractor for compensation over and above the agreed-upon contract amount for additional work or damages supposedly resulting from events that were not included in the initial contract" (Adrian (1993)). This well-known definition implies that contractors can and should ask for a compensation when they execute works that were not considered in the initial contract.[6] Thomas (2001) argued that variations to the work are almost inevitable and Dyer and Kagel (1996) went even further when they stated that - inevitably (sic) - situations arise where clients actually deviate from the original construction scope, which means that, most likely, the initial scope will be increased. These statements strongly sustain our argument that, at least frequently, contractors do end up executing more work than the one deriving from what is established in the tender documents. Consequently, both statements also support the argument that contractors do not know, *ex-ante*, the precise amount of work they will be executing throughout the whole construction phase.

Rooke et al. (2004) categorize construction claims in two different types: (i) proactive claims and (ii) reactive claims. Proactive claims are the ones that can be anticipated and, thus, planned for at the bid preparation stage. On the other hand, reactive claims are the ones that can only be recognized in the course of the project itself, in response to unforeseen events. Even though we are aware that reactive claims may have a substantial impact on the value of the project, we exclude them from our model precisely because they are, by definition, unforeseeable, which means that no acceptable estimate can be drawn. Therefore, our model

[5]The pure detection of hidden-value does not necessarily result in the creation of additional value to the project. The project value will only increase if the execution of the extra volume of work originates a profit. We will discuss this important aspect later.

[6]This definition also implies that there are other sources which may raise more income. However and as we have been stressing, we are only concerned with the ones directly associated with the possible execution of more volume of work.

incorporates estimates for those proactive claims that derive from the presence of uncertainty regarding the volume of work to be performed.[7]

3.2.1 Sources of Additional Value

There are two sources of uncertainty that may result in claims through the execution of more volume of work than the one directly deriving from the information contained in the bid package. We classify them as being of two different kinds: (i) extra quantities and (ii) additional orders.

3.2.1.1 Extra Quantities

Extra quantities occur when the contractor ends up executing, in the field, more quantities of a specific item than the ones specified in the tender documents. As we will see, if the type of contract allows such, the contractor will receive the unit price included in his or her proposal multiplied by the quantities he or she has actually executed and after being measured in the field by the client or the client's agent.[8] Under these contractual conditions, field quantities are the quantities that matter because they are the ones that will generate the income associated with the execution of each task included in the bid package. Ideally, from the client's point of view, field quantities should match the quantities included in the tender documents. However, frequently, discrepancies between the quantities estimated by the client and quantities actually executed in the field are observed. The literature refers that this inaccuracy is mainly due to the poor quality of the tender documents (see, for example, Laryea (2011); Rooke et al. (2004); Akintoye and Fitzgerald (2000)), meaning that the client's estimates are not always accurate and, therefore, tender documents provided to the bidders often contain mistakes.

Bearing this in mind, most experienced contractors do not take for granted the accuracy of the information contained in the tender documents regarding the quantities to be performed when they are preparing the bid. On the contrary, if hidden-value is to be captured and quantified - since inaccuracies in the tender documents are likely to occur - mistakes can only be recognized if a proper measurement of all the technical drawings is performed. This

[7]As we will carefully explain, depending on the contractual arrangements binding the parties, there is a specific type of volume uncertainty which does not necessarily generate more income.

[8]Such fact also implies that the contractor will receive less income if the quantities executed in the field are smaller than the ones specified in the bid documents.

is an important aspect we must stress: contractors will only know, with a strong degree of certainty, how many quantities they will be executing during the project's life cycle if a thorough and accurate measurement of all the technical drawings included in the bid package is undertaken. Moreover, as Rooke et al. (2004) stated, pricing a tender involves reading through bills of quantities often several inches thick, meaning that the quantities stated in the bill must be confronted with the quantities obtained after performing a complete examination of all the drawings provided by the client. Rooke et al. (2004) also argued that, most of the times - especially in the case of non-large contractors - companies do not have experts in this type of highly skilled job or, if they do, the amount of work in hands in a particular moment may imply the need for hiring external experts. This aspect is reinforced by the fact that contractors actually express concern over what they consider to be a short period of time that is normally allowed for bid preparation, as Laryea and Hughes (2008) observed.

3.2.1.2 Additional Orders

Additional orders, also known as "change orders", refer to a task or a set of tasks the contractor effectively performs during the project's life cycle and that possess a different nature from the ones specified in the bid package. This source of uncertainty that may give rise to additional work and extra profit is, thus, different from the one mentioned before, since change orders are related with varied work which is not of a similar character, or is not carried out under similar conditions than the one contained in the bid package (Davinson (2003)).[9] However, we need to make clear that these tasks may include, for the purposes of their completion, the execution of an item or a set of items that actually were considered in the bill of quantities and previously priced by the bidder, since they were part of the project's initial scope. Hence, when contractors look for mistakes in the tender documents, they do not focus their attention merely in finding discrepancies that may lead to the execution of extra quantities solely associated with the tasks specified in the bid package. Instead, experienced engineers and skilled experts also search for possible tasks, which are likely to be executed and were not specified in the tender documents. By carefully analyzing all the plans and drawings provided by the client, it is possible to recognize that some parts of the project (or even the project "seen" as a whole) will not be properly completed if only the tasks included in the tender documents are to be performed. Hence, additional orders can and should be considered as a potential source

[9] Change orders are also often designated as "increase in scope". The designation acknowledges the fact that the scope of the original project becomes wider, which means that the contractor will end up executing a certain number of tasks that were not included in the technical pieces that support the initial contract.

of additional value and our model will consider this argument by assuming that contractors are able to stipulate a high-value estimate and a low-value estimate for the amount of additional orders, and also attribute, to each of these estimates, a probability of occurrence. For the purpose of accurately defining the two estimates, we argue that contractors need to take into account: i) the amount of work the additional orders will generate in comparison to the amount established in the original contract; ii) the previous experience with the client as well as their history and frequency of placing new orders; iii) the bargaining skills of the client throughout the negotiation process.

3.2.2 Types of Contract

To fully understand the possible impact of the two sources of additional value on the project's final profit, we have to relate each of them with the type of contract that will bind the parties. Construction management literature addresses with more relevance two types of contracts (see, for example, Halpin and Senior (2011); Clough et al. (2000); Woodward (1997)): (i) the "unit-price" type of contract and (ii) the "lump-sum" type of contract.[10]

The unit-price contract allows for flexibility in meeting variations regarding the amount and quantity of work encountered during the construction stage. This means that, when this type of contract is adopted, the project is broken down into work items, which are characterized by units, such as cubic yards, linear and square feet, and piece numbers (Halpin and Senior (2011)). This fact implies that the contractor, during the bid preparation stage, will quote the price by units rather than as a single total contract price. Hence, if for some reason, the contractor effectively executes more quantities of one or more specific items included in the tender documents, he or she will be receiving the amount that results from multiplying the number of units executed by the unit price he or she has included in the bid proposal.

If the type of contract enforced is the lump-sum, bidders are asked to price a specific task or item, regardless of the number of units that will actually be executed. Hence, if this type of contract is adopted, contractors will never receive more (less) income for executing more (less) quantities of an item or items clearly specified by the client than those he or she has predicted after analyzing the drawings and other technical documents contained in the bid package. The risk associated with the likelihood of performing more quantities than those

[10]Other types of contract are mentioned in the literature, as the "cost-plus-fee" type and the "cost-reimbursement" type. However, unit-price and, especially, lump-sum contracts are the ones that are most commonly adopted, particularly in the context of bidding competitions.

that served as the basis for computing the corresponding global price for a specific task (or a group of tasks) is, thus, borne by the contractor. However, the opposite may also occur: contractors might actually perform less quantities in the field than those considered during the bid preparation process, and which served as the basis for establishing the proposed bid price. Hence, and even though this specific type of uncertainty still exists when the parties are bounded by a lump-sum contract, it is not possible to account for its effects during the bid preparation stage since the contractor will only be aware if any additional value is actually raised through this mean after the task or tasks in question are executed, *i.e.*, as the project unfolds. Therefore, this source of uncertainty may affect the project value but can not be quantified before the project is initiated. Being so, in the presence of a lump-sum contract, additional value may only be obtained through the execution of additional orders whereas, if the contract assumes the unit-price type, both sources of uncertainty may create additional value by increasing the volume of work to be performed.[11]

Lump-sum contracts are the most common type of contracting, especially in the building sector (Rooke et al. (2004)). In the European Union, current legislation concerning public contracting virtually imposes the lump-sum form, which means that the unit-price type has had small to none application due to the increasing effort european regulators have been exercising with the purpose of transferring the risk associated with possible mistakes (also referred to, in technical language, as "errors and omissions") encountered in the technical pieces from the client to the contractor. This broad reality has compelled us to consider in our model only one of the two sources of additional value previously described: the additional value that may rise from the execution of additional orders. Thus, we will assume that the lump-sum type of contract is the one that actually binds the parties, which means that the possible execution of more quantities in the field than the ones eventually stated in the bid documents will not generate any additional revenues and, consequently, any additional value to the project. This also implies that the costs associated with the possible execution of any extra quantities should be taken into account when determining the amount of constructions costs that will sustain the bid price.

[11] Some projects encompass both types of contracts, which means that some tasks should be priced using unit-prices and others applying the lump-sum form. In such cases, both types of uncertainty are present, in different parts of the project.

3.3 How the Detection of Hidden-Value May Lead to Additional Value: Contractor's Opportunistic Bidding Behavior

Construction management literature clearly acknowledges the fact that the construction industry features strong levels of price competitiveness (see, for example, Chao and Liu (2007); Skitmore (2002); Ngai et al. (2002)) which may force bidders to lower their profit margin and, hence, increase the probability of winning the contract (Mohamed et al. (2011)). Consequently, it is not rare to see the winning bid include a near-zero profit margin (Chao and Liu (2007)) or even a price below-cost. This intense competition encountered in bidding processes often leads to "under-pricing", a common phenomenon namely explained by the need for work and penetration strategies (Yiu and Tam (2006); Fayek (1998); Drew and Skitmore (1997)).[12]

In fact, contractors realize that bidding low when facing strong competition increases the chance of being selected to execute the project but they are also aware of the opposite: if the profit margin included in their proposals is higher, the probability of getting the contract will be lower. This inverse relationship between the level of the profit margin and the probability of winning the bid is a generally accepted fact both in the construction industry and in the research community (*e.g.*, Christodoulou (2010); Kim and Reinschmidt (2006); Tenah and Coulter (1999); Wallwork (1999)). As we will detail, our model incorporates this crucial element and a mathematical expression that respects the inverse relationship between these two variables is adopted.

Detecting hidden-value and executing more volume of work will only result in more value to the project if the difference between the extra revenues and the extra costs of performing the additional tasks is positive, *i.e.*, if contractors do actually generate a profit by executing them, which means that detecting and executing more volume of work than the one directly specified in the tender documents will not necessarily lead to more profit. However, experienced contractors that capture hidden-value ensure themselves that items where extra quantities are likely to be executed will be priced in a way that will lead to an increase in the project value. By applying this practice during the bid preparation stage, contractors increase their probability of winning the bid by sacrificing the profit margin included in the bid proposal, knowing

[12] Under-pricing is not necessarily the same as bidding below-cost; rather, we interpret this concept as the practice of including in the bid price a profit margin lower than the one contractors would include in normal circumstances, *i.e.*, if the levels of price competition in the construction industry were not generally perceived as being particularly intense.

that they may recover (at least) a part of the profit in subsequent change orders or claims, as Tan et al. (2008) observed. This type of behavior is designated in the literature as "Opportunistic Bidding Behavior" (OBB).[13] Thus, following a proactive approach and assuming that time is actually invested in detecting mistakes that may lead to the likely execution of more (less) quantities, contractors will inflate (deflate) the unit price of the items where those mistakes were spotted. Over-charging (under-charging) those items can then be compensated by under-charging (over-charging) the unit-prices of some of the items whose quantities contractors are certain to be accurately measured. Hence, this compensation mechanism allows contractors to maintain the previously defined overall price for executing the quantities specified in the tender documents and, still, leaving room for generating more profit through the likely execution of additional quantities.

Despite the fact that this behavior is potentially more effective in the presence of a unit-price form of contract, it may also produce positive effects when the type of contract enforced is the lump-sum. In fact, experienced contractors will most likely inflate prices of items they predict to be present in future additional orders since - and even though additional orders are subject to a specific process of price negotiation - it is likely that they will contain the execution of certain items which were considered in the original contract and, hence, whose price is already established between the parties. In these circumstances, the parties will agree that the unit price for such items will be the same. However, items that are different from the ones contained in the tender documents become a matter of negotiation between the contractor and the client or the client's agent, as Dyer and Kagel (1996) stated. This means that, unlike what happens with extra quantities, there is no predetermined form of pricing additional orders in its full extension. In fact, contractors do not have a way of predicting, with complete certainty, what price will be established and what profit will be generated if these additional orders are placed by the client.

Nevertheless, based on previous experiences and in current market prices, we believe that contractors can actually perform fair estimates on the final revenues to be generated by these additional orders and we also believe that, in the event such orders are placed and the addi-

[13] The literature identifies two different types of OBB: "front loading" and "claim loading" (see, for example, Arditi and Chotibhongs (2009); Yiu and Tam (2006)). Front loading consists in over-charging the tasks to be executed in the early stages of the project's life cycle and compensate such effect by under-charging the tasks to be performed in the last stages. We are not concerned with this type of behavior in our model since it does not derive from any detection of hidden-value.

tional work executed, a considerable profit will be made.[14]

The remainder of the present Chapter unfolds as follows. In Section 3.4 we introduce the model's basic numerical solution that we have thoroughly described in Chapter II, and which will enable us to reach the optimal price if no detection and quantification of hidden-value is considered. After listing the assumptions in Section 3.5, we proceed to describe the model in Section 3.6. In Section 3.7, a numerical example is given, followed by a sensitivity analysis to some of the model's most important parameters. Finally, in Section 3.8, conclusions and remarks are given.

3.4 The Model

3.4.1 Introduction

The model herein suggested is based on the option to sign the contract and, consequently, to invest in performing the project by the selected bidder. In a bidding competition, the contractor prepares the bid proposal and submits it until a certain date previously defined by the client. However, the client will only decide which bidder will be invited to sign the contract months later. Consequently, the estimated constructions costs that served as the basis to establish the price included in the bid proposal will most likely vary during this period, *i.e.*, from the moment the bid proposal is closed until the selected bidder is invited to sign the contract. On the contrary, the price established by the contractor and proposed to the client will remain unchanged during the same period. Recognizing these facts, we have identified, in Chapter II, a specific real option: the option to sign the contract and, hence, to invest in performing the project by the selected bidder. This option constitutes a real option since the selected bidder has the right - but not the obligation - to sign the contract and, consequently, to invest in executing the job by incurring in the necessary costs to complete it - the construction costs. As the option pricing theory states, this real option has value and, in Chapter II, we used an adapted version of the Margrabe (1978) exchange option pricing model to evaluate this option, with the final purpose of reaching an optimal price. According to the model proposed in Chapter II, the optimal price will be the one corresponding to the highest value of

[14]Dyer and Kagel (1996) conducted a study where a number of general contractors were interviewed. Additional orders were frequently mentioned as being particularly profitable. It is generally accepted in the industry that the negotiation process leading to the price determination of new orders often develops in a very favorable manner to the contractor, mainly due to the client's awareness that the decision to switch to another contractor for merely executing those additional tasks will imply incurring in side costs and will also cause time delays.

the option to invest, weighted by the probability of winning the bid, since the option can only be exercised by the selected bidder. We then proposed a mathematical relationship linking the level of the profit margin (or the "mark-up", as it is commonly designated in construction parlance) and the probability of winning the contract. This mathematical expression respects the generally accepted fact that there is an inverse relationship between the two variables, as we previously mentioned. In fact, even though construction managers seldom support their mark-up decisions using some sort of mathematical expression linking the price and the probability of winning the bid, which means that the decision regarding the mark-up bid value is generally sustained in subjective judgments, gut feelings and heuristics (Hartono and Yap (2011)), managers have, at least, an implicit perception that the higher (lower) the profit margin he or she includes in the bid proposal the lower (higher) will be the probability of winning the bid.

Being so, the numerical solution we proposed in Chapter II has two components: (i) the value of the option to sign the contract and, consequently, to invest in executing the project; (ii) the probability of winning the bid. We briefly present each of them again below.

3.4.1.1 The Value of Option to Sign the Contract and Invest in Performing the Project in the Presence of Penalty Costs

In Chapter II, we discussed that, in some legal environments, a financial compensation may be imposed to the selected contractor if he or she declines the invitation to sign the contract. In fact, and according to Halpin and Senior (2011), in the United States contractors are free to withdraw their bids without incurring in any penalties if that happens prior to the ending of the bidding period. However, if the selected bidder decides to withdraw the proposal after that moment, a penalty equal to the difference between the second best proposal and the chosen bid may be legally imposed, even if the contract has not yet been signed. To accommodate these circumstances, we adapted the Margrabe (1978) exchange option pricing formula. Let P_g denote the bid price when penalty costs are considered. The value of the option to invest, $F_g(P_g, K)$ in the presence of penalty costs, g will be given by the following equation:[15]

$$F_g(P_g, K) = [P_g e^{-r(T-t)} N(d_1) - KN(d_2) - gK(1 - N(d_2))] \tag{3.1}$$

[15]Penalty costs may also be seen as possessing a reputational nature, as we have mentioned.

being (d_1) and (d_2):

$$d_1 = \frac{ln[P_g/(1-g)K-(r-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{3.2}$$

$$d_2 = d_1-(\sigma\sqrt{T-t}) \tag{3.3}$$

where K designates the construction costs, $N(d_1)$ and $N(d_2)$ are the probability density functions for the values that result from expressions (d_1) and (d_2), respectively, r is the risk-free interest rate, σ is the standard deviation and σ^2 is the variance, which, in this case, equals σ_K^2.[16] $T-t$ is the 'time to expiration', *i.e.*, the time between the moment the bid price is established and the moment the contractor is invited to sign the contract, and g is the expected value for the penalty costs. As explained in the previous Chapter, we will assume that the value of the penalty costs, g is a percentage of K, the construction costs.[17]

3.4.1.2 The Probability of Winning the Bid

In Chapter II, we proposed an inverse relationship linking the mark-up ratio and the probability of winning the bid, given by the following equation:

$$W(P,K) = e^{-b(P/K)^n} \tag{3.4}$$

where $W(P,K)$ is the probability of winning the bid, P/K is the mark-up ratio and 'n' and 'b' are parameters included in the equation for calibration purposes. As we have argued in Chapter II, these parameters should be calibrated in order to best reflect each contractor's specific circumstances and the conditions surrounding the project in hands. Parameter 'b'

[16]Since $\sigma^2=\sigma_P^2-2\sigma_P\sigma_K\rho_{PK}+\sigma_K^2$, where ρ_{PK} is the correlation coefficient between the price, P and the construction costs, K; since P remains unchanged during the life of the option, then σ_P^2 equals zero and so does $2\sigma_P\sigma_K\rho_{PK}$.

[17]In the absence of penalty costs, g equals zero and, thus, the Margrabe (1978) formula is reduced to the original form: $F(P,K)=[Pe^{-r(T-t)}N(d_1)-KN(d_2)]$, where:

$$d_1 = \frac{ln(P/K)-(r-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

and d_2 remains unchanged.

sets the probability of winning the bid if the price equals the construction costs, *i.e.*, if the mark-up ratio equals 1. Parameter 'n' is responsible for shaping the function's concavity and convexity. Our model embraces this functional relationship and a specific calibration for each of the parameters will be specified in the numerical example presented in Section 3.7.

3.4.1.3 The Base Price

We designate the "base price", in the context of the present model, as being the optimal price that results from the maximization problem proposed in Chapter II. We will adopt that numerical solution to determine this price. The base price is the price contractors should include in their bid proposal if no detection and quantification of hidden-value that may lead to the generation of additional profit through the execution of additional orders is to be undertaken, in accordance with the assumptions listed below in Section 3.5. Hence, the base price is the optimal price without considering those effects or, which is the same, the optimal price that derives from considering the value solely generated through the execution of the tasks included in the bid package.

Assuming that penalty costs, g are present, let P_b designate the base price. P_b will be the outcome of the maximization problem:

$$\begin{aligned} V(P_b,K_b) \quad = \max_{P_b} \quad & \{[(P_b e^{-r(T-t)})N(d_1) - KN(d_2) - gK_b(1-N(d_2))] \\ & [e^{-b(P_b/K_b)^n}]\} \end{aligned} \tag{3.5}$$

where K_b is the amount of costs the contractor will have to incur in order to exclusively perform the amount of work specified in the initial contract, *i.e.*, the base construction costs. Thus, the option value for each mark-up level, $V_g(P_b,K_b)$ will be given by the outcome of the adapted Margrabe (1978) formula, $F_g(P_b,K_b)$ weighted by the probability of winning the bid, $W(P_b,K_b)$.

3.5 Assumptions

We will assume that (i) each bidder decides what price to bid without engaging in any kind of interaction or contact with other bid participants; (ii) each bidder prepares his or her proposal simultaneously with the other competitors; (iii) each bidder presents a single-sealed proposal to the client; (iv) each bidder has access to the information contained in the bid package, allowing him or her to establish the base price, determined applying the maximization problem given by equation (3.5); (v) the selected bidder will only decide if he or she is going to perform the project when the contract has to be signed and not before that date; (vi) it is possible to establish a mathematical relationship between the mark-up level and the probability of winning the bid and the expression linking these two variables is given by equation (3.4); (vii) penalty costs are present if the selected contractor decides not to sign the contract and, consequently, not invest in executing the project; (viii) the parties are bound by a lump-sum contract, which means that no additional income is generated if extra quantities are executed; (ix) by only using the skills of their own experienced staff, contractors are able to stipulate a high-value estimate and a low-value estimate for the expected profit to be generated through the execution of additional orders and also to attribute a probability of occurrence to each of them, during the bid preparation stage; (x) once the true estimate is known, contractors will adjust the price (extra revenues) during the negotiation process to compensate for any variations that may occur in the estimated extra costs, which means that any changes observed in the necessary costs to perform the additional orders will lead to an adjustment in the price requested to the client, with the purpose of maintaining the expected profit at the level previously established during the bid preparation period.

3.6 Model Description

3.6.1 Introduction

The present model is motivated by the fact that, in most construction projects, the volume of work to be executed is not known with precision during the bid preparation stage. Hence, uncertainty is present concerning the level of profit to be generated. Even though we have identified two different sources of volume uncertainty, we will only focus on the one deriving from the possible execution of additional orders to be placed by the client, since we are assuming that the parties are bound by a lump-sum contract - thus preventing contractors from generating extra revenues by merely executing extra quantities of items included in the bid package.

As previously mentioned, we will consider that experienced contractors are be able to stipulate a high-value estimate and a low-value estimate to the additional orders and to attribute a probability of occurrence to each of them just by undertaking a preliminary assessment of the tender documents, which means that this goal can be achieved without the need to incur in any additional costs associated with hiring skilled professionals and contracting specialized firms, *i.e.*, without any incremental investment in human capital and technology. For the sake of simplicity, we will refer to the mere allocation of working time of the persons possessing the necessary skills to perform these tasks (*e.g.*, engineers and estimators) as "non-incremental investment". Let C_1 designate the level of this non-incremental investment that contractors will undertake using only the skills of their own experienced staff. By investing the amount C_1, contractors will thus (i) define a high-value estimate and a low-value estimate for the price (revenues) to be obtained through the execution of additional orders; (ii) stipulate a high-value estimate and a low-value estimate for the necessary costs to successfully perform these orders; (iii) attribute a probability of occurrence to each of the estimates.

Therefore, by investing the amount C_1, contractors will be establishing a discrete-time stochastic variable, designated as "additional value", with two possible outcomes, and affecting a probability of occurrence to each of them.

3.6.2 The Impact of the Non-Incremental Investment

The base price, P_b represents the amount of income to be received due to the execution of the volume of work included in the bid package: this is the price resulting from expression (3.5) presented above and - again - we stress that this is the optimal price contractors should include in their proposals if no detection and quantification of any additional value deriving from the execution of additional orders is undertaken. By investing the amount C_1, contractors will most likely detect and quantify hidden-value, which may result in the creation of additional value to the project. Hence, we first need to consider the additional income that will be received, assuming that additional orders will be executed during the project's life cycle and, secondly, the necessary costs to successfully perform the additional work. Being so, let p_A represent the additional income (extra revenues) that derives from the possible execution of additional orders, and k_A the amount of costs the contractor will have to incur in order to perform these new orders. Finally, π represents the amount of profit (or additional value) generated by executing the additional orders, *i.e.*, the difference between p_A and k_A.

Also, let (i) p_A^H designate the high-value estimate for the revenues associated with the execution of the additional orders; (ii) p_A^L designate the low-value estimate for such revenues; (iii) k_A^H designate the high-value estimate for the costs associated with the execution of the additional orders; (iv) k_A^L designate the low-value estimate for such costs; (v) θ represent the probability associated with p_A^H and k_A^H; hence $(1-\theta)$ is the probability associated with p_A^L and k_A^L. Finally, let π^H and π^L denote the additional profit for the high-value and the low-value estimates, respectively. π^H and π^L will be given by the following equations:

$$\pi^H = p_A^H - k_A^H \tag{3.6}$$

$$\pi^L = p_A^L - k_A^L \tag{3.7}$$

Thus, the expected value for the additional profit, $E(\pi)$ will be given by equation (3.8):

$$E(\pi) = \pi^H\theta + \pi^L(1-\theta) \tag{3.8}$$

Let P_1 designate the optimal price in the present conditions, *i.e.,* the price that incorporates the effect of the expected value for the additional profit, $E(\pi)$, given by equation (3.8). Hence, we adapt equation (3.5) with the purpose of incorporating the effects caused by the expected value for the additional profit, $E(\pi)$. Let P_1^* denote the optimal price according to these conditions. Hence, P_1^* will be the outcome for the following maximization problem:

$$\begin{aligned} V(P_1, K_b) = \max_{P_1}\{[((P_1+E(\pi))e^{-r(T-t)})N(d_1) - K_bN(d_2) - \\ gK_b(1-N(d_2))][e^{-b(P_1/K_b)^n}]\} \end{aligned} \tag{3.9}$$

The optimal price, P_1^* that results from the maximization problem given by equation (3.9) is smaller than the one resulting from the maximization problem given by equation (3.5), *i.e.*, the base price, P_b since the former is the optimal price in the absence of any recognition and quantification of hidden-value generating more profit through the execution of additional orders, whereas the latter reflects the optimal price considering the expected impact of the additional orders to be performed, at this stage, by investing the amount C_1.[18] This means that P_1^* is the price contractors should include in their bid proposals because it is the optimal

[18]This implies that the expected value for the additional profit, $E(\pi)$ is greater than zero.

price if no incremental investment is undertaken.

We also need to make clear that equation (3.9) is a function of K_b - the direct costs of solely performing the tasks included in the tender documents - rather than the total estimated costs the contractor will incur if he or she wins the contract and performs the project. This is due to the fact that the extra costs are already considered in the amount of the expected profit which, as the same equation shows, integrates the maximization problem given by equation (3.9).

Being so, the probability of winning the contract adopting the optimal price, P_1^* will given by the following equation:

$$W(P_1^*, K_b) = e^{-b(P_1^*/K_b)^n} \tag{3.10}$$

The outcome of equation (3.10) is greater than the outcome of equation (3.4). In fact, the probability of winning the contract considering the effects of investing C_1 will always be greater than the probability considering the base price, P_b, because P_1^* - and assuming that some hidden-value is captured and quantified at this stage - is smaller than the base price, P_b, *i.e.,* the optimal price if no quantification of hidden-value is considered, as we have mentioned. Thus, just by investing the amount C_1, contractors will produce a more competitive bid price, provided that some hidden-value leading to the generation of additional profit through the execution of additional orders has actually been captured and quantified.

3.6.2.1 The Value of the Option to Sign the Contract and Perform the Project

Considering the effects of the expected additional profit, $E(\pi)$, the value of the option to invest, $V(P_1, K_b)$ will be given by the following equation:

$$\begin{aligned} V(P_1^*, K_b) = \{[(P_1^* + E(\pi))e^{-r(T-t)}N(d_1) - K_b N(d_2) - \\ gK_b(1 - N(d_2))][e^{-b(P_1^*/K_b)^n}]\} \end{aligned} \tag{3.11}$$

being:

$$d_1 = \frac{ln[(P_1^* + E(\pi))/(1-g)K_b] - (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{3.12}$$

and

$$d_2 = d_1 - (\sigma\sqrt{T-t}) \tag{3.13}$$

$N(d_1)$ and $N(d_2)$ the probability density functions for the values resulting from expressions (d_1) and (d_2), respectively, r is the risk-free interest rate, σ is the standard deviation and σ^2 is the variance which, as we have mentioned, equals σ_K^2. $T-t$ is the 'time to expiration', *i.e.*, the time between the moment the price included in the bid proposal is established and the moment the contractor is invited to sign the contract and, finally, g is the estimated level of penalty costs.

Equation (3.11) determines the value of the option to invest if the expected value for the additional profit to be generated by executing the additional orders, $E(\pi)$ is, in fact, the true value. Thus, if no incremental investment is put through, the outcome of equation (3.11) is the value of the option to sign the contract and perform the project after the amount C_1 has been invested and assuming that some hidden-value, resulting in more volume of work, was captured and quantified as a direct result of such investment.

3.6.3 The Impact of the Incremental Investment in Human Capital and Technology

As we mentioned previously, construction companies often have to invest in human capital and technology and, hence, hire skilled technicians and/or highly specialized firms possessing the necessary know-how and technology to perform specific studies, whose purpose is to supply managers with more accurate information concerning the project in hands, during the bid preparation stage. Let C_2 denote the value of this incremental investment, which will allow the contractor to eliminate the uncertainty concerning the true value of the additional work to be performed and the extra profit to be generated through the execution of such additional work. Hence, after investing the amount C_2, the contractor may face two different scenarios since this investment will reveal if the true value is the high estimate or the low estimate, previously defined. Being so, we first need to determine the optimal price and the corresponding value of the option to invest, for each of the scenarios.

3.6.3.1 The Optimal Price and the Value of the Option to Invest Considering the High-Value Estimate

If the investment in C_2 reveals that the true value for the additional profit, π is given by the high-estimate, then the optimal price in these conditions, $P_2^{*^H}$ will be the outcome of the maximization problem given by equation (3.14):

$$
\begin{aligned}
V(P_2^H, K_b) \quad = \quad & \max_{P_2^H}\{[(P_2^H + \pi^H)e^{-r(T-t)}N(d_1) - KN(d_2) - \\
& gK_b(1 - N(d_2))][e^{-b(P_2^H/K_b)^n}]\}
\end{aligned} \tag{3.14}
$$

where:

$$
d_1 \quad = \quad \frac{ln[(P_2^H + \pi^H)/(1-g)K_b] - (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{3.15}
$$

and

$$
d_2 \quad = \quad d_1 - (\sigma\sqrt{T-t}) \tag{3.16}
$$

The value of the option to invest, assuming that the true value for the additional profit equals the high-value estimate, is given by the following equation:

$$
\begin{aligned}
V(P_2^{*^H}, K_b) \quad = \quad & [(P_2^{*^H} + \pi^H)e^{-(T-t)}N(d_1) - K_bN(d_2) - \\
& gK_b(1 - N(d_2))][e^{-b(P_2^{*^H}/K_b)^n}]
\end{aligned} \tag{3.17}
$$

3.6.3.2 The Optimal Price and the Value of the Option to Invest Considering the Low-Value Estimate

However, if the true value revealed for π is given by the low-estimate, then the optimal price, $P_2^{*^L}$ will be the outcome of the following maximization problem:

$$
\begin{aligned}
V(P_2^L, K_b) &= \max_{P_2^L}\{[(P_2^L + \pi^L)e^{-r(T-t)}N(d_1) - K_bN(d_2) - \\
&\quad gK_b(1 - N(d_2))][e^{-b(P_2^L/K_b)^n}]\}
\end{aligned} \tag{3.18}
$$

And the value of the option to invest, in these conditions, will be given by equation (3.19):

$$
\begin{aligned}
V(P_2^{*^L}, K_b) &= [(P_2^{*^L} + \pi^L)e^{-r(T-t)}N(d_1) - K_bN(d_2) - \\
&\quad gK_b(1 - N(d_2))][e^{-b(P_2^{*^L}/K_b)^n}]
\end{aligned} \tag{3.19}
$$

being:

$$
d_1 = \frac{ln[(P_2^{*^L} + \pi^L)/(1-g)K_b] - (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{3.20}
$$

and

$$
d_2 = d_1 - (\sigma\sqrt{T-t}) \tag{3.21}
$$

3.6.4 The Threshold Value for the Incremental Investment

By (i) weighting the outcome of equation (3.17) by the probability associated with the high-value estimate; (ii) weighting the outcome of equation (3.19) by the probability associated with the low-estimate and (iii) adding up these two results, we reach the value of the option to invest considering the two scenarios, $V(P_2^*, Kb)$. Hence:

$$
V(P_2^*, K_b) = \theta V(P_2^{*^H}, K_b) + (1-\theta)V(P_2^{*^L}, K_b) \tag{3.22}
$$

The outcome of equation (3.11) is the value of the option to invest considering solely the effects produced by the non-incremental investment, C_1, *i.e.*, $V(P_1^*, K_b)$. Equation (3.22) determines the value of the option considering the effects of the incremental investment,

C_2, considering both scenarios together, each weighted by the corresponding probability of occurrence. Hence, the difference between the outcome of equation (3.22) and the outcome of equation (3.11) is the exact amount of the incremental investment, C_2, below which any level of incremental investment will add value to the project. Let C_2^* denote this threshold value for the incremental investment. C_2^* will thus be given by the following equation:

$$C_2^* = V(P_2^*, K_b) - V(P_1^*, K_b) \tag{3.23}$$

and the ratio between the incremental investment threshold and the base construction costs is as follows:

$$RC_2^* = \frac{C_2^*}{K_b} \tag{3.24}$$

3.7 Numerical Example

3.7.1 The Base Case

The following table includes information about the inputs used in the present numerical example:

Table 3.1: inputs: description and values

Inputs	**Description**	**Values**
K_b	base construction costs	€ 100,000,000
σ	standard deviation	0.25
r	risk-free interest rate	0.01
$T-t$	"time to expiration"	0.5 (years)
n	calibration parameter of the function W(P,K)	10
b	calibration parameter of the function W(P,K)	ln(1/0.5)
g	penalty costs	0.02
p_A^H	high-value estimate for the additional revenues	€ 30,000,000
k_A^H	high-value estimate for the additional costs	€ 15,000,000
p_A^L	low-value estimate for the additional revenues	€ 9,000,000
k_A^L	low-value estimate for the additional costs	€ 6,000,000
θ	probability of occurrence of the high-value estimate	0.5
$(1-\theta)$	probability of occurrence of the low-value estimate	0.5

Using the inputs listed above and applying the model described in Section 3.6, we have reached the following outputs:

Table 3.2: outputs: description, values and corresponding equations

Outputs	Description	Values	Equation
P_b	base price	€ 101,942,247	(3.5)
π^H	additional profit considering the high-value estimate	€ 15,000,000	(3.6)
π^L	additional profit considering the low-value estimate	€ 3,000,000	(3.7)
$E(\pi)$	expected value for the additional profit	€ 9,000,000	(3.8)
P_1^*	optimal price after investing C_1	€ 99,000,845	(3.9)
$W(P_1^*, K_b)$	probability of winning with price P_1^*	53,42%	(3.10)
$V(P_1^*, K_b)$	option value after investing C_1	€ 6,091,160	(3.11)
$P_2^{*^H}$	optimal price considering the high-value estimate	€ 97,250,537	(3.14)
$\theta V(P_2^{*^H}, K_b)$	option value considering the high-value estimate	€ 4,325,005	(3.17)
$P_2^{*^L}$	optimal price considering the low-value estimate	€ 100,911,754	(3.18)
$(1-\theta)V(P_2^{*^L}, K_b)$	option value considering the low-value estimate	€ 2,012,839	(3.19)
$V(P_2^*, K_b)$	option value considering both estimates	€ 6,337,844	(3.22)
C_2^*	incremental investment threshold value	€ 246,684	(3.23)
RC_2^*	investment threshold / base construction costs	0.247%	(3.24)

3.7.2 Sensitivity Analysis

3.7.2.1 Is There a Scale-Effect?

Assuming that the project dimension is given by the value of the base construction costs, K_b, we performed a sensitivity analysis with the purpose of verifying if a scale-effect is present, *i.e.*, if the investment threshold ratio, RC_2^* assumes different values in response to variations in the amount of the base construction costs. We defined two alternative scenarios, where (i) the amount of the base construction costs are twice as great and four times as great as in the base case, *i.e.*, equal to € $200,000,000$ and € $400,000,000$, (ii) the level for the high-value estimate and the low-value estimate of the additional profit respects this same proportion and (iii) the corresponding probabilities of occurrence remain unchanged. We reached the following results for the incremental investment threshold value, C_2^* and for the incremental investment threshold ratio, RC_2^*:

Table 3.3: sensitivity analysis: scale-effect

(for: $\sigma = 0.25$; r =0.01; $T - t = 0.5$; $n = 10$; $b = ln(1/0.5)$; $g = 0.02$)

	base case	**alternative scenario 1**	**alternative scenario 2**
K_b	**€ 100,000,000**	**€ 200,000,000**	**€ 400,000,000**
π^H	€ 15,000,000	€ 30,000,000	€ 60,000,000
π^L	€ 3,000,000	€ 6,000,000	€ 12,000,000
θ	50%	50%	50%
$(1-\theta)$	50%	50%	50%
$E(\pi)$	€ 9,000,000	€ 18,000,000	€ 36,000,000
C_2^*	€ 246,684	€ 493,368	€ 986,736
RC_2^*	**0.247%**	**0.247%**	**0.247%**

The results included in Table 3.3 clearly demonstrate that the incremental investment threshold value is proportional to the amount of the base construction costs, which means that no scale-effect is present. In fact, for the three dimensions considered, the ratio between the incremental investment threshold and the base construction costs remains constant and equal to 0.247%, which means that there is a linear relationship between them.

3.7.2.2 The Impact of Variations in the Probabilities Associated with the High/Low Value Estimates

The results concerning the impact of considering different levels for the probabilities associated with the high-value and the low-value estimates on the model's outcome are included in Table 3.4.

Table 3.4: sensitivity analysis: probabilities associated with high/low value estimates

(for information about the inputs used, please refer to Table 3.1)

θ	$(1-\theta)$	$E(\pi)$	$V(P_1^*, K_b)$	$V(P_2^*, K_b)$	C_2^*	RC_2^*
99%	1%	€ 14,880,000	€ 8,594,122	€ 8,603,767	€ 9,645	0.01%
90%	10%	€ 13,800,000	€ 8,009,638	€ 8,187,667	€ 88,029	0.09%
80%	20%	€ 12,600,000	€ 7,568,344	€ 7,725,143	€ 156,799	0.16%
70%	30%	€ 11,400,000	€ 7,056,369	€ 7,262,710	€ 206,341	0.21%
60%	40%	€ 10,200,000	€ 6,563,919	€ 6,800,277	€ 236,358	0.24%
50%	**50%**	**€ 9,000,000**	**€ 6,091,160**	**€ 6,337,844**	**€ 246,684**	**0.247%**
40%	60%	€ 7,800,000	€ 5,638,219	€ 5,875,410	€ 237,191	0.24%
30%	70%	€ 6,600,000	€ 5,205,182	€ 5,412,977	€ 207,795	0.21%
20%	80%	€ 5,400,000	€ 4,792,090	€ 4,950,544	€ 158,453	0.16%
10%	90%	€ 4,200,000	€ 4,398,939	€ 4,488,110	€ 89,171	0.09%
1%	99%	€ 3,120,000	€ 4,062,110	€ 4,071,920	€ 9,810	0.01%

The results included in Table 3.4 clearly show that, the closer the probabilities are to the upper limit or the lower limit, the smaller is the investment threshold value. The explanation resides in the fact that, the closer the probabilities are to 100% or to 0%, the lower is the uncertainty regarding which will be the true value, meaning that the incremental investment assumes now a lower importance in resolving this uncertainty. The two more extreme scenarios clearly reflect this: when parameter θ equals 99% or 1%, the investment threshold assumes very low values (€ $9,645$ and € $9,810$, respectively). On the contrary, as probabilities tend to 50%, the higher is the threshold value, C_2^*. Thus, the incremental investment threshold reaches the maximum value when the level of uncertainty is the highest, *i.e.*, when the probabilities associated with the two estimates are the same.

3.7.2.3 The Impact of Variations in the Difference Between the Two Estimates

Table 3.5 includes values concerning three different scenarios and the results reached by changing the difference between the high-value and low-value estimates but assuming that both the expected profit, $E(\pi)$ and the probabilities of occurrence, θ and $(1-\theta)$ remain unchanged.

Table 3.5: sensitivity analysis: difference between the high and the low value estimates

(for: $\sigma = 0.25$; $r = 0.01$; $T-t = 0.5$; $n = 10$; $b = ln(1/0.5)$; $g = 0.02$)

	base case	**alternative scenario 1**	**alternative scenario 2**
K_b	€ 100,000,000	€ 100,000,000	€ 100,000,000
π^H	€ 15,000,000	€ 17,000,000	€ 13,000,000
π^L	€ 3,000,000	€ 1,000,000	€ 5,000,000
$(\pi^H - \pi^L)$	**€ 12,000,000**	**€ 16,000,000**	**€ 8,000,000**
θ	50%	50%	50%
$E(\pi)$	€ 9,000,000	€ 9,000,000	€ 9,000,000
$V(P_1^*, K_b)$	€ 6,091,160	€ 6,091,160	€ 6,091,160
$V(P_2^*, K_b)$	€ 6,337,844	€ 6,528,210	€ 6,201,066
C_2^*	**€ 246,684**	**€ 437,050**	**€ 109,906**
RC_2^*	**0.247%**	**0.437%**	**0.110%**

In the alternative scenario 1, the difference between the two estimates is greater than in the base case. The results show that, when this difference increases from € $12,000,000$ (the difference in the base case) to € $16,000,000$, the incremental investment threshold also increases (from € $246,684$ to € $437,050$). The explanation resides in the fact that contractors face more uncertainty concerning which of the two estimates will become the true value and, hence, the incremental investment assumes a higher importance in resolving such uncertainty. In fact, the corresponding threshold value is greater since the increase in the value of the option to invest considering both estimates, $V(P_2^*, K_b)$ assumes now a higher value, whereas the value of the option to invest considering solely the effects of the non-incremental investment, $V(P_1^*, K_b)$ remains unchanged. On the contrary, for a smaller difference between the two estimates (€ $8,000,000$), as the results reached for the alternative scenario 2 clearly reflect, the investment threshold is smaller: €$109,906$, compared to € $246,684$, in the base case. The level of uncertainty associated with the two estimates is now lower and this lower level of uncertainty is reflected in the value of option to invest considering both estimates, $V(P_2^*, K_b)$.

The value of this option is, in the alternative scenario 2, smaller than in the other two cases and, thus, closer to the value of the option to invest considering only the investment in C_1, $V(P_1^*, K_b)$, whose value does not depend upon the differences between the two estimates, since the expected value for the additional profit, $E(\pi)$ remains unchanged.

We thus conclude that, the higher (lower) the difference between the high-value estimate and the low-value estimate, the higher (lower) will be the uncertainty concerning which estimate will become the true value, and the higher (lower) will be the value of the option to invest considering the two estimates, $V(P_2, K_b)$. Consequently - since the value of the option to invest considering only the effects of the non-incremental investment, $V(P_1, K_b)$ remains unchanged - the greater (smaller) will be the value for investment threshold, C_2^*, and the greater (smaller) will be the value of the ratio RC_2^*.

3.8 Conclusions and Remarks

Several types of uncertainty surround construction projects and construction managers should proactively manage the effects they may produce in the project value. We approached a specific type of uncertainty and designated it as “volume uncertainty”. This type of uncertainty is critical since, at least frequently, managers do not know with precision the amount of work they will be executing throughout the project’s life cycle and, consequently, the expected final profit the project will generate. To assess the impact of volume uncertainty on the value of the project, we defined a discrete-time stochastic variable and designated it as “additional value”. Additional value is the value that is hidden in the the most uncertain parts of the project and, in the context of the present research, is defined as the one that does not derive from merely executing the tasks specified in the bid package.

To capture and quantify this type of value, construction companies need to invest. Initially, by merely applying the skills of his or her own experienced staff, construction managers are able to define a high-value estimate and a low-value estimate for the additional profit and to stipulate a probability of occurrence to each of the estimates. Based on the numerical solution proposed in Chapter II, we suggested a model which determines that managers will produce a more competitive bid even if no incremental investment is undertaken, provided that some hidden-value is captured and quantified during the bid preparation stage. However, in order to resolve the uncertainty concerning which of the two estimates will become the true value for the expected additional profit, contractors often need to invest in human capital and technology and, thus, hire specialized firms and highly skilled professionals. The model's outcome is the threshold value for this incremental investment. Therefore, managers may use a simple decision rule: hire external services with the purpose of eliminating the uncertainty concerning which of the two estimates previously established is the true value, provided that the cost of this incremental investment in human capital and technology does not exceed the threshold value previously determined. Any amount paid for external services, which is lower than the threshold value, will lead to an increase the project value and, the lower this cost, the higher will be the increase in the project value. On the contrary, if the amount actually invested exceeds the predetermined threshold value, the value of the project will be reduced. The model also determines the optimal bid price in the case no incremental investment in human capital and technology is undertaken, in the case the true value reached by undertaking the incremental investment equals the high-estimate for the additional value and also in the case the true value equals the low-estimate for the additional value, both previously stipulated.

Sensitivity analysis showed that no scale-effect is present in the model since the incremental investment threshold value responds linearly to variations in the project dimension. Sensitivity analysis also showed that, the closer to 50% is the probability of occurrence associated with the estimates, the greater the threshold value is since undertaking the incremental investment assumes a higher importance due to the presence of higher levels of uncertainty concerning which of the two estimates will become the true value. Finally, sensitivity analysis performed to the difference between the two estimates established for the additional value demonstrated that, the greater the difference between the two estimates the higher the level of uncertainty concerning which of them will become the true value. As a result, the incremental investment assumes a greater importance in eliminating this uncertainty and this greater importance is reflected in a higher threshold value for the incremental investment. On the contrary, if the difference between the two estimates is smaller, the level of uncertainty present is lower, which means that the incremental investment assumes now a smaller importance in resolving this uncertainty and, as a consequence, the incremental investment threshold assumes a lower value.

Chapter 4

A Two-Factor Uncertainty Model to Determine the Optimal Contractual Penalty for a Build-Own-Transfer Project

4.1 Introduction

4.1.1 Build-Operate-Transfer (BOT) Projects in the Context of Bidding Competitions

Public-Private Partnerships (PPP) became one of the most important types of public procurement arrangements and its importance has been growing considerably in the last decades (Kwak et al. (2009); Alonso-Conde et al. (2007); Algarni et al. (2007); Ho and Liu (2002)). PPP is usually defined as a long-term development and service contract between the government and a private partner (Maskin and Tirole (2008)). This type of contract may assume different forms and the Build-Own-Transfer (BOT) model is widely adopted (Liu and Cheah (2009); Ho and Liu (2002)). BOT is the terminology for a project structure that uses private investment to undertake the infrastructure development, and which has been historically ensured by the public sector.[1] In fact, the private sector has been playing an increasingly crucial

[1]The acronym BOT is often used interchangeably with BOOT (build-own-operate-transfer). Other arrangements include BOO (build-own-operate), a type of contractual scheme where the private party does not carry the obligation of transferring the ownership in any date, meaning that the private partner may operate the facility forever, as in the case of a typical private investment project.

role in the financing and provision of services that were traditionally in the domain of the public sector. One of the key reasons is that governments are unable to cope with the ever-increasing demands on their budgets. Most infrastructure expenditures in developing - and also in developed countries - have been funded directly from fiscal budgets but several factors such as the macroeconomic instability and growing investment needs have shown that public finance is volatile and, in many of those countries, rarely meet the infrastructure expenditure requirements in a timely and adequate manner (Ferreira and Khatami (1996)).

In BOT projects, a private entity is given a concession to build an infrastructure and operate a facility which, as we just mentioned, would traditionally be built and operated by the government or a public entity (Shen and Wu (2005)). At the end of the predetermined concession period (a period that tends to be long), the private party returns the ownership of the infrastructure to the government or the public entity (Shen et al. (2002)).[2]

BOT infrastructure projects also differ significantly from the construction projects we have considered in Chapters II and III in how they are implemented during the pre-construction phase. In the case of construction projects, the public party is responsible for project planning, property acquisition and, more importantly, project funding. In BOT projects, the concessionaire (private party) is usually required to undertake these project development tasks (Huang and Chou (2006)). This contractual arrangement provides a mechanism for using private finance, hence allowing the public sector to construct more infrastructure services without the use of additional public funds (Shen and Wu (2005)) .

BOT projects are awarded through an appropriate bidding competition process, where a number of private entities compete to win the contract. As with the other type of construction projects - and all other things being equal - the contract will be awarded to the bidder that presented the most competitive bid, which, in the case of BOT projects, means the bidder that offered the highest price.[3] The price is the amount to be paid to the government to own the right to operate the facility once the obligation of constructing the infrastructure is fulfilled. Hence, the selected bidder will be invited to sign the contract and - if the contract is actually signed - he or she will have to invest in constructing the facility and run the subsequent operations, once the construction phase is completed.

[2]Given that BOT/BOOT schemes are designed and implemented as Public-Private Partnership contractual arrangements, we will consider, for the purpose of the present work, BOT and PPP designations as being synonyms.

[3]In construction projects - and all other things being equal - the selected bidder is the one that presented the lowest bid. For obvious reasons, in a BOT project competitive bid, is the other way around: all else equal, the selected bidder will be the one that offered the highest price.

4.1.2 The Real Options Approach in the BOT Projects Literature

A considerable number of research pieces can be found where the real options approach is applied to address different questions concerning BOT projects. Some of the most important issues concern project valuation, how several risk types should be shared between the parties, the role of incentives and subsidies given by the government or the public entity and also the well-known mechanism of the Minimum Revenue Guarantee (MRG). By addressing these topics using the real options approach, researchers seem to acknowledge that a number of real options are available to both parties throughout the life of the project. Thus, flexibility is a feature that can be frequently found on existing contracts and the levels of uncertainty surrounding this type of projects tend to be high. By recognizing the irreversibility of this type of investments and also the uncertainty and flexibility that characterize BOT projects, some researchers adopt the real options methodology to evaluate options embedded in current contract types, while others use it to sustain and propose different forms of shaping contracts between the parties, namely by suggesting that incorporating more flexibility in the contractual relationship may lead to more economic efficiency at different levels. Various research pieces can be found focusing on one or more of these research topics. For example, Huang and Chou (2006) valued the MRG and the option to abandon the project by the private firm, and Brandao and Saraiva (2008) also developed a model for infrastructure projects based on the consideration of a Minimum Demand Guarantee. Cheah and Liu (2006) addressed the valuation of demand and revenue guarantees, applying Monte Carlo simulation, and Chiara et al. (2007) also applied the same technique to evaluate a MRG which is only redeemable at specific moments in time. Alonso-Conde et al. (2007) addressed the existing contractual conditions in PPP which guarantee a minimum profitability to the private firm. Shan et al. (2010) proposed collar options (a call option and a put option combined) to better manage revenue risks. Huang and Pi (2009) applied a sequential compound option approach for valuing multi-stage BOT projects, in the presence of dedicated assets. Caselli et al. (2009) valued the indemnification provision that ensures a final compensation to the private partner, in the event the government terminates a BOT contract. Armada et al. (2012) showed how the net cost of incentives, which may be given by the government to induce immediate investment, should equal the value of the option to defer the beginning of operations by the private firm. Pereira et al. (2006) applied a two-factor uncertainty model aiming to reach the optimal timing for the construction of an international airport, where the cash flows to be generated were disaggregated into the number of passengers and the net cash flow “per passenger”. Their model is thus based on the existence of two stochastic variables, but they considered the investment costs constant.

4.1.3 Two-Factor Uncertainty Models in the Real Options Literature

All of the above research pieces do not consider the two key-value drivers of a BOT project - the construction costs and cash flows to be generated by operations - as stochastic variables. To the best of our knowledge, the only research piece based on a two-factor uncertainty model, in the context of BOT projects, where the stochastic variables are the present value of the cash flows and the construction costs is the paper by Ho and Liu (2002). In fact, these researchers considered the construction costs and the present value of cash flows as being both stochastic variables, behaving according to geometric Brownian motions. However, their model is designed in discrete-time, more specifically applying the well-known binomial model, with the purpose of valuing a debt guarantee given by the government, and also accounting for the risk of bankruptcy. The model we propose and describe in Section 4.3 is a two-factor uncertainty model in continuous-time where both construction costs and cash flows follow geometric Brownian motions that are possibly correlated and, to the best of our knowledge, is the first model based on a two-factor uncertainty approach in continuous-time, in the context of BOT projects.

In fact, the consideration of both key-value drivers of BOT projects as stochastic variables is not a common feature in the existing research. However, we should stress that, in recent years, two-factor uncertainty models have been increasingly adopted to address other research topics in the field of real options, such as the research pieces carried out by Armada et al. (2013), Adkins and Paxson (2011) and Paxson and Pinto (2005)

The traditional real options model on the optimal timing to invest in a project with irreversible costs and generating perpetual cash flows was first developed by McDonald and Siegel (1986), where both the present value of the cash flows and the investment costs are considered to behave stochastically. This model is carefully explained in Dixit and Pindyck (1994). These authors describe the model but held the investment costs constant in a first stage. Later in their text book, Dixit and Pindyck (1994) address the bi-dimensionality issue - thus recognizing the stochastic nature of the investment costs - and present McDonald and Siegel (1986) solution, which consists of reducing the two stochastic variables to just one, by merely substituting the cash flows and investment costs by a single variable (ratio) that equals the cash flows divided by the investment costs. McDonald and Siegel (1986) model is also described in Trigeorgis (1996). Both Trigeorgis (1996) and Dixit and Pindyck (1994) stated that it is optimal to invest when the ratio between the cash flows and the investment costs reaches a given boundary: the so-called "free boundary", separating the waiting region from the investing region. This method may be very useful to address some BOT projects related issues but, unfortunately, we cannot apply it to all situations in the context of the present work, since we can not invoke homogeneity of degree one in all the boundary conditions, unlike what happens in McDonald and Siegel (1986) model.[4]

The model we suggest in the present Chapter is based on the existence of uncertainty in both the facility construction costs and in the value of the cash flows that will be generated by operating the project facility. Hence, a two-factor uncertainty approach is adopted, with both variables following geometric Brownian motions that are possibly correlated. In the next Section, we detail the proposed framework where the government grants leeway to the selected firm regarding the timing for project implementation, which means that the private firm may implement the project when he or she decides that is optimal to do so. However, we suggest that a contractual penalty should be enforced in the event the private firm does not implement the project immediately. The theoretical model we propose aims to determine the optimal level for this contractual penalty and will consider the argument that the selected firm is more efficient than the government in executing the project facility and also that the government recognizes the existence of social costs, *i.e.*, the costs that correspond to the loss of social welfare which emerge if the project is not implemented immediately. If both entities are equally efficient, then the optimal penalty would be the one that would induce the selected firm to invest in the same moment the government would, if the government decided to undertake the project. However, we will demonstrate that - by assuming that the private firm is more efficient than the government in executing the project facility - the optimal contractual penalty is the one that makes the private firm invest when his or her value for the cash flows trigger equals the value of the government cash flows trigger, for a given project dimension and assuming a specific level of comparative efficiency. In fact, by considering that the private firm is more efficient, then one expects the private firm to invest sooner than the government would since he or she will attain his or her construction costs trigger sooner than the government will attain its construction costs trigger. By attaining the construction costs trigger sooner than the government and since the optimal contractual penalty is the one that makes both entities have the same trigger for the cash flows, then one expects the private firm to invest sooner than the government would, if the government decided to conduct the

[4]Since their boundary conditions do not infringe homogeneity of degree one, the model proposed by these researchers respect the condition which states that the sum of the roots is equal to 1. A closed-form solution is thus reached.

project. In fact, by introducing the higher efficiency of the private firm in the model, we can not define the optimal penalty as being the one that makes both entities have the same combination of triggers, *i.e.*, expectedly, to invest in same moment of time. Rather, in the presence of the private firm's greater efficiency, the optimal penalty should be defined as the one that moves the private firm cash flows trigger downwards in order to meet the government cash flows trigger, for a given project dimension and considering the estimated value for the comparative efficiency factor.

4.2 Proposed Framework

4.2.1 Flexibility May Imply a Cost. The Contractual Penalty

We define a conceptual framework where a BOT project may be undertaken by the government or awarded to a private firm through an appropriate competitive bid process, maybe because the government recognizes that is less efficient than the private firm in conducting the project. We suggest that the project may be initiated whenever the selected bidder decides it is optimal to do so, meaning that no contractual obligation for immediate initiation of activities is imposed by the government. This assumption regarding the absence of any contractual obligation regarding the immediate implementation of the project has also been adopted by Armada et al. (2012), and plays a crucial role in the context of the research questions they address. The motivation of their research work lies on the argument that the private firm will start investing later than the government would like to, since the option to defer the project implementation does have value. Bearing this in mind, they studied how certain subsidies and guarantees, granted to the private firm, can be optimally arranged with the purpose of inducing the immediate implementation of the project. We build the present framework on the same assumption but argue that, under certain conditions, a legal penalty should be enforced in the event the construction of the facility does not start immediately. Thus, and even though we acknowledge that the private firm may manage the project implementation as far as its initiation/completion is concerned, we suggest that a penalty should be enforced in the case the infrastructure is not ready and operations do not start immediately. This means that the private firm is aware of the fact that delaying the project implementation may grant him or her some benefits (there is value to waiting for more information) but is also aware that deferring the beginning of the project may entail a cost. Thus, the private firm has flexibility regarding the moment to start running operations but this flexibility may imply a cost. We propose that, under certain conditions, which we will explain in this Section, this cost should be considered in the contract form, assuming the form of a legal penalty.

4.2.2 The Comparative Efficiency Factor

We have to acknowledge that governments are, in most cases, less efficient than private firms in conducting BOT projects. This argument is common in the literature and is frequently invoked as being one of the reasons why governments actually grant the projects to the private sector (see, for example, Brandao and Saraiva (2008); Zhang and Kumaraswamy (2001)). Being so, one should expect the private firm to invest earlier due to its greater efficiency. To be more efficient means being able to construct the facility investing less money than the government would. Hence, if the expected constructions costs of the private firm are lower than the construction costs estimated by the government, then - all else equal - the private firm will attain the critical value for the cash flows to be generated by operations that triggers the investment sooner than the government. This is the same as saying that one expects the private firm to invest in an earlier moment of time than the government would.

Hence, by considering that the private firm is more efficient than the government, we are assuming that the private firm will have a lower trigger for his or her construction costs than if both entities were considered to be equally efficient because, if both entities were equally efficient, then they would have the same construction costs trigger. This greater efficiency leads us to conclude that the efficiency factor places the firm construction costs trigger before the government construction costs trigger, "vis-a-vis" with the scenario where both entities are equally efficient. Furthermore, the greater the level of comparative efficiency, the lower the private firm construction costs trigger will be or, which is the same, the further backwards its construction costs trigger will be shifted, again when compared with the "equally efficient scenario".

Considering the private firm's greater efficiency, the government's purpose is accomplished if the private firm invests when his or her cash flows trigger equals the government cash flows trigger. We would like to underline this argument: by enforcing an optimal legal penalty, the government's purpose is to induce the private firm to invest when he or she attains a cash flows trigger whose value equals the value of the government cash flows trigger. This implies that, without the enforcement of the legal penalty, the private firm cash flows trigger may have a higher value than the government cash flows trigger. We will show under which conditions this inequality holds. One of these conditions derives from the fact that the government cash flows trigger is affected by another element that the suggested model considers. We designate this element as "social costs".

4.2.3 Social Costs

The traditional concept of "social cost" was addressed by Coase (1960) and is based on the concept of "externality", in the sense that, by producing a certain good, a harmful effect is caused to the society, rather than the owners of the firm responsible for the production of that good or its customers. It is a cost which society must ultimately bear. This naturally implies that a loss of welfare to the population emerges due to the existence of this externality. Hence, this loss of welfare is not a direct consequence of the fact that a firm is not producing a good or providing a service to the customers. Rather, social cost is exactly defined as being the cost borne by the population because a certain good is being produced and such fact causes an undesirable effect to the population. Notwithstanding, we reason that, when a government or a public entity decides to implement a project, its goal is to provide a service to the population. The government decision of, say, implementing a High-Speed Rail service is based on the conviction that the project will generate a social benefit, as stated by Rus and Nombela (2007). Hence, the government believes that social benefits occur as soon as the project is implemented and operations start because social welfare will emerge once the project is completed. Bearing this important argument in mind, we state that, on the contrary, if the project is not immediately implemented, a loss of welfare occurs and this is the same as stating that the social benefits emerging from the immediate project implementation are postponed until the project is actually completed and the subsequent activities start. We thus argue that the lack of social benefits from the moment the project should have been ready and the moment the project is, in fact, ready and operations begin can be legitimately defined as "social costs".[5] Social costs are, therefore, directly related with the loss of welfare that emerges if the project is not implemented immediately. Again, these costs correspond to the loss of social welfare occurred from the time the project should have been ready to start operating and providing services to its users and the moment operations actually begin and the users needs start being satisfied. Being so, our model considers the existence of social costs in the case the project is not promptly implemented.

[5] We would like to underline the argument that our definition of "social costs" differs from the definition of "social cost", as in Coase (1960). This means that we are not using this concept in the context of the present work because we are not considering the importance of the emergence of social cost, which results from some kind of externality.

Social costs, as we define them, are rarely negligible. BOT projects are undertaken because governments believe that public needs must be satisfied and such needs will only be satisfied once the project is completed and subsequent operations begin. The level of social costs may be high for some projects, moderate for other projects or even low in fewer cases.[6] We will consider that social costs are estimated by the government as being a percentage of the expected cash flows to be generated by operations. By estimating an expected level of social costs, the government is setting its own cash flows trigger or, which is the same, is defining the optimal moment where the project would be initiated if the government decided to undertake the project. Nevertheless, when defining the moment where the project would be implemented, the government behaves as a rational agent, in the sense that also considers the benefits of waiting and invest in a later date (meaning that the government recognizes that there is value to waiting, regardless of which entity will conduct the project) and compares such benefits with the fact that, the later the project is implemented, the higher the value of social costs. Thus, when the government sets its own cash flows trigger, by defining the level of social costs considered to be tolerable, the government takes into account the fact that waiting for better information does have value. The government cash flows trigger will thus result from a trade-off between the benefits of waiting and, hence, not invest immediately and the level of social costs to be borne by the population, if the project is not promptly initiated. Being so, when establishing the level of social costs considered to be acceptable, the government also takes into account the benefits that derive from delaying the project implementation. Consequently, the greater the level of social costs the government considers to be acceptable, the sooner the government would invest. On the contrary, for lower levels of acceptable social costs, the government will invest later. We will numerically demonstrate the existence of this relationship in Section 4.3.

4.2.4 The Optimal Value for the Contractual Penalty

Social costs do play a fundamental role in establishing the optimal value for the contractual penalty to be enforced in the event the project is not immediately implemented. The greater the level of social costs the sooner the government would invest and the higher the optimal penalty needs to be, with the purpose of moving the cash flows trigger downwards to meet the

[6] Currently in Portugal, there is a stream of opinion that questions the benefits of some large-scale investments undertaken in the recent past, especially the construction of highways linking the cities of Lisbon and Porto, which are considered by many to be redundant. In such cases and following this line of thought, delaying the initiation of large-scale investments whose social benefits are perceived as being very low will entail a very low level of social costs.

government cash flows trigger, which is now placed more below. On the contrary, the lower the level of social costs the smaller the optimal penalty needs to be (since the government cash flows trigger is now placed more above) and, if the level of social costs is zero, then a legal penalty is not needed, which is the same as saying that the optimal value is zero. Hence, the optimal value for the contractual penalty is the one that moves the private firm cash flows trigger downwards with the purpose of perfectly meeting the government cash flows trigger, assuming that the private firm cash flows trigger is greater than the government cash flows trigger. Bearing this definition in mind, we have to conclude that any value for the contractual penalty that does not move the private firm cash flows trigger downwards in order to perfectly meet the government cash flows trigger will never be optimal. In Section 4.4, we will discuss the effects, to the government, that derive from including a non-optimal value for the legal penalty in the contract form.

The remainder of the present Chapter unfolds as follows. In Section 4.3, the model is described and both the government decision to invest and the government expectation about the private firm decision to invest are presented. The optimal level for the contractual penalty "per unit" of cost is then reached, considering the effect of the estimated efficiency factor and also the effect caused by the level of social costs. We proceed to perform a sensitivity analysis, aiming to assess the impact of variations in various parameters of the model on the optimal contractual penalty. More specifically, we measure how the optimal value for the contractual penalty is affected by (i) changes in the level of social costs; (ii) changes in the efficiency factor; (iii) variations in both the level of social costs and the level of the comparative efficiency factor. Furthermore, we derive the analytical solution to the level of the efficiency factor above which the inclusion of legal penalty is not justified, for a given level of social costs. The same solution also enables us to determine the level of social costs, for a given comparative efficiency factor, above which a contractual penalty should be enforced. Still in Section 4.3, we assess the impact of changes in the correlation coefficients and in the standard deviations of both variables. In Section 4.4, we use a numerical example in order to demonstrate the effects, to the government, of including a non-optimal value for the legal penalty in the contract from, as a result of overestimating or underestimating the comparative efficiency factor. Finally, in Section 4.5, concluding remarks are given.

4.3 The Model

4.3.1 Assumptions

We assume that (i) the government or other public entities have the necessary know-how to determine fair estimates about the construction costs and the cash flows to be generated through operating the facility, as if the project was conducted by the government; (ii) the standard deviations of the facility construction costs and of the future cash flows are the same for both entities since variations in the value of the project inputs and the project outputs are both observable in the market; (iii) the rate of return-shortfalls (*i.e.,* the "convenience yields") are also the same for both entities; (iv) the government recognizes that is less efficient in constructing the project facility than the private firm; (v) the government is able to determine an estimate for the private firm's greater level of efficiency; (vi) the construction of the facility is instantaneous;[7] (vii) the project, once implemented, will generate perpetual cash flows;[8] (viii) the government acknowledges the emergence of social costs if the project is not completed and operations do not start immediately; (ix) social costs are estimated as a percentage of the present value of the cash flows to be generated by operations; (x) the project dimension is given by the expected amount of construction costs estimated by the government.

4.3.2 Model Description

4.3.2.1 Introduction

We depart from considering that the various sources of uncertainty underlying a BOT project may be reduced to two aggregated sources: the uncertainty regarding the value of the future cash flows to be generated by operations, V, and the facility construction costs, K. In specific situations, we will follow Adkins and Paxson (2011) approach, also based on a two-factor uncertainty model, and consisting of a set of simultaneous equations. These authors developed a quasi-analytical solution, whereby a boundary between the continuance and the renewal

[7] By assuming that the construction of the facility is instantaneous, we are implicitly considering that no flexibility is present throughout the construction period. Therefore, we exclude the existence of options during this stage. Since flexibility is not considered, we are only concerned with the fact that construction costs are uncertain.

[8] We assume that the concession period is sufficiently long for the investment opportunity to be considered equivalent to a perpetual *call* option. This is a common assumption in the literature. See, for example, Armada et al. (2012).

regions (*i.e.*, the boundary separating the regions where values for both variables justify an incumbent asset to continue operating or to be renewed and, thus, substituted by a new one by paying a fixed cost) is determined, since they are unable to invoke homogeneity of degree one in the boundary conditions. Likewise, some of the questions we propose to answer can not be addressed by invoking homogeneity of degree one, hence leading us to follow Adkins and Paxson (2011) approach. By considering that a trade-off between the two stochastic variables is present and also that only when both triggers are simultaneously attained the investment is triggered, Adkins and Paxson (2011) state that a countless set of pairs for the triggers do exist, which means that there is an infinite combination of possible threshold values for both variables. In our model, it is possible to envisage a countless set of pairs of threshold values for the stochastic variables, V^* and K^* and also that this countless set of pairs defines the discriminatory boundary separating the waiting region from the investing region. A variation in one of the variables leads to a variation of the same sign in the other for the investment to be triggered. In fact, a lower value for the construction costs trigger will necessarily lead to a lower value for the cash flows to trigger the investment and, similarly, a higher value for the construction costs trigger will lead to a higher cash flows trigger value in order to prompt the investment. As Dixit and Pindyck (1994) point out, the value of the option to invest depends on both variables, meaning that one would expect the option to be exercised when V becomes sufficiently high for a given K or K becomes sufficiently low for a given V. On the contrary, if V is not sufficiently high for a given K or K is too low for a given V, then one would expect the option to be held. Hence, we are aware that is possible to define an optimal boundary composed by a set of pairs for V^* and K^* that discriminates the waiting region from the investing region. We will denote these pairs of trigger values as $\{V^*, K^*\}$.

In the present Section, we proceed to present the base case parameter values and derive the government decision to invest, bearing in mind that social costs are present and considered to be a percentage of the cash flows to be generated by operations. This implies that the government recognizes the fact that, if the project is not immediately implemented, a loss of social welfare occurs until the project is completed and operations actually start. We then derive the government expectation about the private firm investment decision, assuming the private firm is more efficient than the government in executing the project facility. Finally, we determine the optimal value for the contractual penalty "per unit" of cost, and consider three different project dimensions, for illustrative purposes.

For both cases, *i.e.*, the government decision to invest and the expectation the government has about the private firm's decision to invest, the two stochastic variables (the value of the cash flows to be generated by operations, V, and the facility construction costs, K) follow geometric Brownian motions that are possibly correlated. Let $i \in \{G, P\}$, where G stands for government and P stands for the government expectation about the private firm. Being so:

$$dV_i = \alpha_{V_i} V_i dt + \sigma_{V_i} V_i dz_{V_i} \tag{4.1}$$

$$dK_i = \alpha_{K_i} K_i dt + \sigma_{K_i} K_i dz_{K_i} \tag{4.2}$$

$$E(dz_{V_i}, dz_{K_i}) = \rho_i dt \tag{4.3}$$

where α_{V_i} and α_{K_i} are the drift parameters, dt is the time interval, σ_{V_i} and σ_{K_i} the standard deviations for each of the variables, dz_{V_i} and dz_{K_i} the corresponding increments of standard Wiener processes and, finally, ρ_i is the correlation coefficient between V_i and K_i.

Table 4.1 includes the base case parameter values we will be using, unless otherwise mentioned.

Table 4.1: the base case parameter values

(for $i \in \{G, P\}$)

Parameters	Symbols	Values
standard deviation of the cash flows	σ_{V_i}	0.15
standard deviation of the construction costs	σ_{K_i}	0.10
cash flows rate of return-shortfall	δ_{V_i}	0.03
construction costs rate of return-shortfall	δ_{K_i}	0.03
correlation coefficient	ρ_i	0
risk-free interest rate	r	0.05
level of social costs	s	0.02
comparative efficiency factor	γ	0.2

4.3.2.2 The Government Decision to Invest

Under risk-neutrality, the value of the government investment opportunity, $F_G(V_G, K_G)$ must satisfy the following partial differential equation (Constantinides, 1978):

$$\frac{1}{2}\sigma_{V_G}^2 V_G^2 \frac{\partial^2 F_G}{\partial V_G^2} + \frac{1}{2}\sigma_{K_G}^2 K_G^2 \frac{\partial^2 F_G}{\partial K_G^2} + \rho \sigma_{V_G} \sigma_{K_G} V_G K_G \frac{\partial^2 F_G}{\partial V_G \partial K_G} + \\ (r - \delta_{V_G}) V_G \frac{\partial F_G}{\partial V_G} + (r - \delta_{K_G}) K_G \frac{\partial F_G}{\partial K_G} - rF_G - sV_G = 0 \tag{4.4}$$

where r is the risk-free interest rate, sV_G denotes the value of social costs as a percentage of the cash flows to be generated by the project operations, δ_{V_G} and δ_{K_G} are the rates of return-shortfall for V_G and K_G, respectively, which are given by the following equations:

$$\delta_{V_G} = r - \alpha_{V_G} \tag{4.5}$$

$$\delta_{K_G} = r - \alpha_{K_G} \tag{4.6}$$

The following general solution satisfies the partial differential equation (4.4):

$$F_G(V_G, K_G) = A_1 V_G^{\beta^+} K_G^{\eta^+} + A_2 V_G^{\beta^+} K_G^{\eta^-} + A_3 V_G^{\beta^-} K_G^{\eta^+} + A_4 V_G^{\beta^-} K_G^{\eta^-} - \frac{s}{\delta_{V_G}} V_G \tag{4.7}$$

where A_1, A_2, A_3 and A_4 are constants that need to be determined. β^+, β^-, η^+ and η^- are the four roots of an elliptical equation, which is the two-factor counterpart of the one-factor stochastic model quadratic equation presented in Dixit and Pindyck (1994). The elliptical equation is:

$$Q(\beta, \eta) = \frac{1}{2}\sigma_{V_G}^2 \beta(\beta - 1) + \frac{1}{2}\sigma_{K_G}^2 \eta(\eta - 1) + \\ \rho_G \sigma_{V_G} \sigma_{K_G} \beta\eta + (r - \delta_{V_G})\beta + (r - \delta_{K_G})\eta - r = 0 \tag{4.8}$$

The function $Q(\beta,\eta) = 0$ defines an ellipse.[9] Equation (4.8) has four quadrants, to which of them corresponds a pair of the four roots just mentioned, *i.e.*, the four points where the elliptic function meets the axes. If we now consider the absorbing barriers $F_G(0,0)$, $F_G(0,K_G)$, thus $A_3 = A_4 = 0$, since the option to invest is worthless if the present value of future cash flows is zero. Likewise, for $F_G(V_G,K_G)$, and as K_G becomes infinitely large for any value of V_G, then the option is also worthless. To respect this condition, we need to set constant $A_1 = 0$, leaving only constant $A_2 \neq 0$. Thus, in our case, the quadrant of interest is the second one, corresponding to the pair of roots $\{\beta^+,\eta^-\}$. The general solution for the homogeneous partial differential equation (4.4), given by equation (4.7) is thus reduced and takes the following form:

$$F_G(V_G,K_G) = A_2 V_G^{\beta^+} K_G^{\eta^-} - \frac{s}{\delta_{V_G}} V_G \tag{4.9}$$

The value matching condition implies that, when the trigger values for both V_G and K_G are simultaneously attained, the value of the option to invest must equal the project's expected Net Present Value (NPV). Being so:

$$F_G(V_G^*,K_G^*) = A_2 {V_G^*}^{\beta^+} {K_G^*}^{\eta^-} - \tfrac{s}{\delta_{V_G}} V_G^* = V_G^* - K_G^* \tag{4.10}$$

As the value-matching condition supports homogeneity of degree one on both sides of the equation, we can reduce the dimensionality of the problem to only one variable, since $\beta^+ + \eta^- = 1$. Thus, the trigger value for V_G^* will be given by the following equation:[10]

$$V_G^* = \frac{\beta}{\beta-1}\frac{K_G^*}{(1+\frac{s}{\delta_{V_G}})} \tag{4.11}$$

where β is the root of the fundamental quadratic equation (4.12), whose value exceeds one.[11]

[9] Please refer to Adkins and Paxson (2011) for more details.

[10] Please note that, in the absence of social costs, the solution given by equation (4.11) is reduced to the one presented in Dixit and Pindyck (1994) for the case of two stochastic variables.

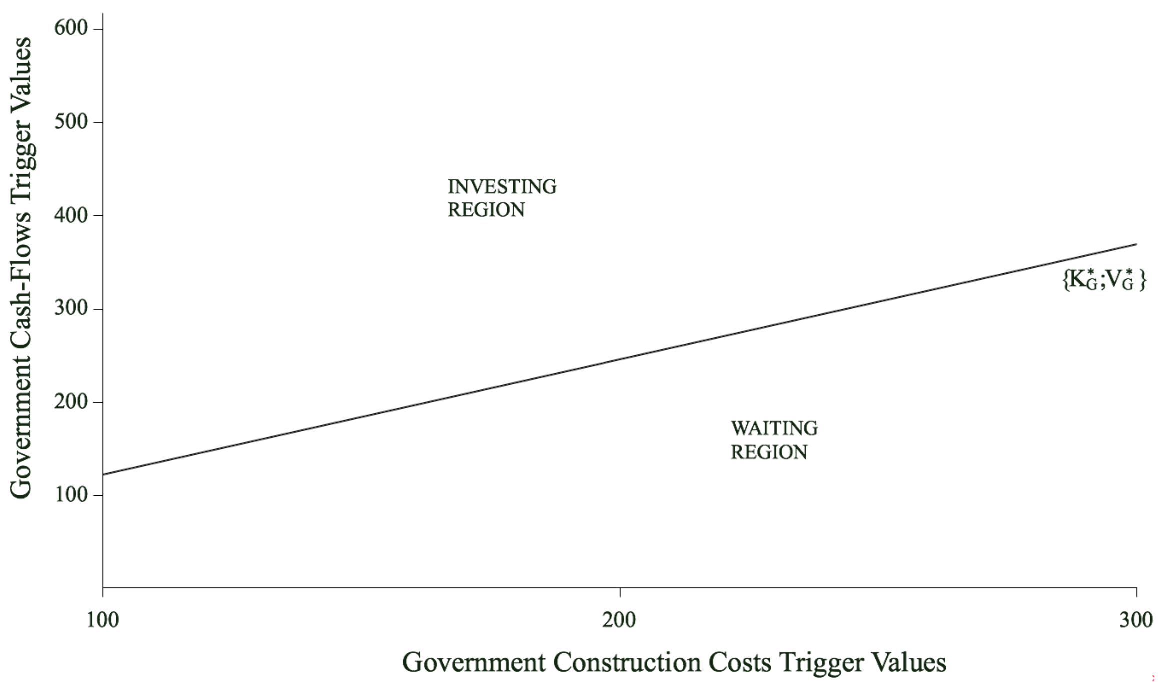
[11] Please refer to Dixit and Pindyck (1994) for further details.

$$Q = \frac{1}{2}(\sigma_{V_G}^2 - 2\rho\sigma_{V_G}\sigma_{K_G} + \sigma_{K_G}^2)\beta(\beta-1) + (\delta_{K_G} - \delta_{V_G})\beta - \delta_{K_G} = 0 \qquad (4.12)$$

Equation (4.11) leads us to conclude that there is a countless pairs of government triggers, $\{K_G^*, V_G^*\}$ whose values depend on the value of the root β, on the level of social costs estimated by the government, s and on cash flows rate of return-shortfall, δ_{V_G}. A discriminatory boundary does exist, separating the waiting region from the investing region, provided that we stipulate a set of values for K_G^* or a set of values for V_G^*. By setting a value for one of the two triggers, we reach the value of the other trigger by merely applying equation (4.11) for a given level of social costs.

Figure 4.1 illustrates the government discriminatory boundary, separating the waiting region from the investing region. We considered a set of pairs, $\{K_G^*; V_G^*\}$, with K_G^* ranging from 100 to 300. The base case parameter values included in Table 4.1 apply.

Figure 4.1: the government discriminatory boundary



The discriminatory boundary in Figure 4.1 has a constant and positive slope (which is given by: $\frac{\beta}{(\beta-1)(1+\frac{s}{\delta_{V_G}})}$) and, thus, the relationship between the two variables is linear. An increase (decrease) in K_G^* of, say, 10%, will lead to an increase (decrease) of 10% in the value of the cash flows for the investment to be triggered. The existence of this relationship means that a trade-off between the cash flows trigger and the construction costs trigger is present. As Dixit and Pindyck (1994) stated, when one of the two triggers is affected by a (negative) positive variation, the other variable needs to respond with a (negative) positive variation of the same dimension in order to trigger the investment.

4.3.2.3 The Government Expectation about the Private Firm Investment Decision

We have been assuming that the private firm is more efficient than the government in constructing the project facility. Being so, let γ denote the comparative efficiency factor. We can relate the government construction costs with the expectation the government has about the private firm construction costs applying the following equation:

$$K_P = K_G(1-\gamma) \tag{4.13}$$

We now proceed to compute the trigger values of the private firm as estimated by the government. The firm may face a penalty cost, which we denote by c, for delaying the project implementation. Let $F_P(V_P, K_P)$ denote the government expectation about the value of the private firm investment opportunity. Under risk-neutrality, $F_P(V_P, K_P)$ must satisfy the following partial differential equation:

$$\begin{aligned} \frac{1}{2}\sigma_{V_P}^2 V_P^2 \frac{\partial^2 F_P}{\partial V_P^2} + \frac{1}{2}\sigma_{K_P}^2 K_P^2 \frac{\partial^2 F_P}{\partial K_P^2} + \rho_P \sigma_{V_P} \sigma_{K_P} V_P K_P \frac{\partial^2 F_P}{\partial V_P \partial K_P} + \\ (r-\delta_{V_P}) V_P \frac{\partial F_P}{\partial V_P} + (r-\delta_{K_P}) K_P \frac{\partial F_P}{\partial K_P} - rF_P - c = 0 \end{aligned} \tag{4.14}$$

Equation (4.14) has a non-homogeneous part, c and the rest of the equation is homogeneous. We reach the general solution for the homogeneous part of the partial differential equation (4.14) by applying the same reasoning as in the previous case*, i.e.,* the government decision to invest. Being so, the general solution for the homogeneous part of equation (4.14), $F_P^H(V_P, K_P)$ takes the form:

$$F_P^H(V_P, K_P) = A_5 V_P^{\beta^+} K_P^{\eta^+} + A_6 V_P^{\beta^+} K_P^{\eta^-} + A_7 V_P^{\beta^-} K_P^{\eta^+} + A_8 V_P^{\beta^-} K_P^{\eta^-} \tag{4.15}$$

As in the previous case, we are only interested in the pair of roots $\{\beta^+, \eta^-\}$, which simplifies the solution given by equation (4.15). Hence, we reach a reduced solution, given by the following equation:

$$F_P^H(V_P, K_P) = A_6 V_P^{\beta^+} K_P^{\eta^-} \tag{4.16}$$

The particular solution for the non-homogeneous part, F_P^{NH} is as follows:

$$F_P^{NH} = -\frac{c}{r} \tag{4.17}$$

Stitching together the homogeneous and non-homogeneous solutions, we obtain the following expression that satisfies the whole partial differential equation (4.14):

$$F_P^H(V_P, K_P) + F_P^{NH} = F_P(V_P, K_P) = A_6 V_P^{\beta^+} K_P^{\eta^-} - \frac{c}{r} \tag{4.18}$$

The value-matching condition, in these circumstances, is given by:

$$F_P(V_P^*, K_P^*) = A_6 V_P^{*\beta^+} K_P^{*\eta^-} - \frac{c}{r} = V_P^* - K_P^* \tag{4.19}$$

Unfortunately, this value-matching condition does not support homogeneity of degree one on both sides, which means that $\beta^+ + \eta^- \neq 1$. To overcome this problem, we follow Adkins and Paxson (2011) quasi-analytical approach by constructing a set of three simultaneous equations. The first equation is the elliptical equation (4.20), which is equivalent to equation (4.8):

$$
\begin{aligned}
Q(\beta,\eta) &= \frac{1}{2}\sigma_{V_P}^2\beta(\beta-1)+\frac{1}{2}\sigma_{K_P}^2\eta(\eta-1)+ \\
&\rho\sigma_{V_P}\sigma_{K_P}\beta\eta+(r-\delta_{V_P})\beta+(r-\delta_{K_P})\eta-r=0
\end{aligned} \tag{4.20}
$$

The other two equations are the two smooth-pasting conditions derived from the value-matching condition, given by equation (4.19).

The first smooth-pasting condition is the first-order derivative of the value-matching condition, in respect to V_P^*. After some rearrangements, we obtain the first smooth-pasting condition, given by the following equation:

$$
\beta^+(V_P^*-K_P^*+\frac{c}{r}) = V_P^* \tag{4.21}
$$

The second smooth-pasting condition is the first-order derivative of the value matching condition, given by equation (4.19), in respect to K_P^*. Likewise, after some rearrangements, we reach the second smooth-pasting condition, given by equation (4.22):

$$
\eta^-(V_P^*-K_P^*+\frac{c}{r}) = -K_P^* \tag{4.22}
$$

Equations (4.20), (4.21) and (4.22) form the set of simultaneous equations which will enable us to define the private firm discriminatory boundary, composed by a set of countless pairs $\{K_P^*,V_P^*\}$ and, therefore, to obtain the private firm's waiting and investing regions in the presence of a given value for the contractual penalty, c. For this purpose - since we face the existence of four unknowns, $V_P^*,K_P^*,\beta^+,\eta^-$ and we only have three equations available - we follow Adkins and Paxson (2011) solution in order to determine the four unknowns. Thus, we first need to set K_P^* and determine the corresponding, β^+,$\eta^-$and V_P^* . Then, by perturbing K_P^* and repeating the process, we gather a set of pairs $\{K_P^*,V_P^*\}$ that define the private firm discriminatory boundary.

4.3.2.4 Determining the Optimal Value for the Contractual Penalty

In order to reach the optimal value for the contractual penalty, we will first consider that $c=0$, *i.e.*, we will determine the private firm cash flows trigger as if no legal penalty was enforced.

We will use the set of three simultaneous equations formed by equations (4.20), (4.21) and (4.22) for this purpose. This enables us to obtain new values for V_P^*, after perturbing K_P^*.[12] Thus, by applying this procedure, we determine the private firm cash flows trigger, for a given K_P^*, as if no penalty was enforced. We will designate, from now on, this cash flows trigger as V_P^+ and the corresponding construction costs trigger as K_P^+.[13] Hence, V_P^+ represents the private firm cash flows trigger that needs to be moved downwards in order to perfectly meet the government cash flows trigger, V_G^*, for any given K_P^+.[14] Secondly, we follow Adkins and Paxson (2011) approach and use the set of three simultaneous equations formed by equations (4.20), (4.21) and (4.22) with the purpose of determining the exact value for c that moves V_P^+ downwards to perfectly meet V_G^*.[15] The value of c that perfectly moves V_P^+ downwards in order to meet V_G^* is the optimal value for the contractual penalty. Hence, the optimal contractual penalty is the one that makes $V_P^* = V_G^*$ for a given K_P^* or, which is the same, the one that perfectly aligns both cash flows triggers. We designate the optimal value for the contractual penalty as c^*.

The following table exhibits a set of representative values for K_G^*, V_G^*, K_P^*, V_P^+, c^* and V_P^*. The values were obtained applying the procedures previously described. We consider three possible project dimensions, given by $K_G^* = 100$, $K_G^* = 200$ and $K_G^* = 300$.

Table 4.2: representative values for the government triggers, the private firm triggers and the optimal contractual penalty, for three project dimensions

(for: $\sigma_{V_G} = \sigma_{V_P} = 0.15$; $\sigma_{K_G} = \sigma_{K_P} = 0.10$; $\delta_{V_G} = \delta_{V_P} = 0.03$; $\delta_{K_G} = \delta_{K_P} = 0.03$; $\rho_G = \rho_P = 0$; $r = 0.05$; $s = 0.02$; $\gamma = 0.2$)

K_G^*	K_P^*	V_G^*	V_P^+	c^*	V_P^*	$c*/K_G^*$	$c*/K_P^*$
100	80	123	164	**0.9611**	123	0.0096	**0.012**
200	160	247	329	**1.9222**	247	0.0096	**0.012**
300	240	370	493	**2.8833**	370	0.0096	**0.012**

The results included in Table 4.2 clearly show that the level of the optimal contractual penalty, c^* is proportional to the level of K_G^* and, consequently, also proportional to the level of K_P^*.

[12]Perturbing K_P^* does not change the value of the roots, β^+, η^- since we are considering, at this stage, that $c = 0$. Thus, $\beta^+ + \eta^- = 1$, for any K_P^*.

[13]$K_P^+ = K_P^*$, as we are fixing K_P^* to obtain V_P^+.

[14]Considering that V_P^+ is greater than V_G^*, otherwise there would be no need for a legal penalty to be included in the contract.

[15]As we will demonstrate, this does not mean that the new discriminatory boundary for the private firm overlaps the government discriminatory boundary.

This means that no scale-effect is present in the model: an increase (decrease) in the construction costs trigger will lead to an increase (decrease) in the value of the optimal legal penalty of the same magnitude. This proportionality feature allows us to determine the level of the optimal contractual penalty "per unit" of the private firm's expected construction costs which, according to the inputs considered, equals approximately 0.012.

Figure 4.2 illustrates the impact of the optimal contractual penalty on the private firm discriminatory boundary. The government discriminatory boundary, formed by pairs $\{K_G^*; V_G^*\}$ is presented again. The private firm discriminatory boundary before considering the impact of the contractual penalty, formed by pairs $\{K_P^+; V_P^+\}$ and the private firm discriminatory boundary, formed by pairs $\{K_P^*; V_P^*\}$ - which results from considering the impact of the optimal contractual penalty "per unit" of the private firm expected construction costs - are both designed, for the three project dimensions.

Figure 4.2: the impact of the optimal contractual penalty on the private firm discriminatory boundary

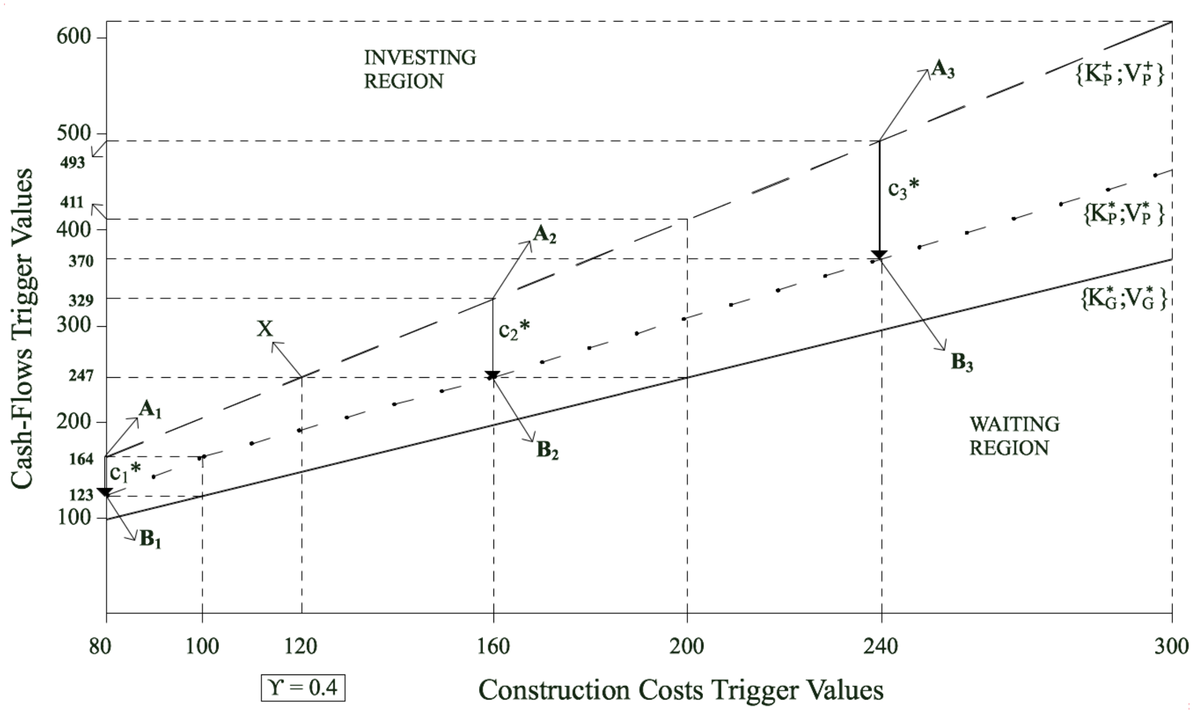


Figure 4.2 shows the private firm pair of triggers before considering the impact of the opti-

mal legal penalty, for the three dimensions. If no legal penalty was enforced in the contract, the government would expect the private firm to have a discriminatory boundary formed by a countless set of trigger pairs, $\{K_P^+; V_P^+\}$ represented by a countless set of points ($A_1 = \{80; 164\}$, $A_2 = \{160; 329\}$ and $A_3 = \{240; 493\}$ are three examples of those points). Solving the system of simultaneous equations composed by equations (4.20), (4.21) and (4.22), we have reached the three points $B_1 = \{80; 123\}$, $B_2 = \{160; 247\}$ and $B_3 = \{240; 370\}$. By joining these three points, we define another discriminatory boundary: the private firm discriminatory boundary after considering the impact of the optimal contractual penalty, formed by pairs $\{K_P^*; V_P^*\}$. In fact, points B_1, B_2 and B_3 are three of a countless set of points representing a countless set of pairs, $\{K_P^*; V_P^*\}$, which define the private firm discriminatory boundary after considering the impact of the contractual penalty. Figure 4.2 also shows that the private firm discriminatory boundary does not match the government discriminatory boundary. In fact, as we have argued, the optimal penalty for each project dimension, c_1^*, c_2^* and c_3^* is the one that moves the private firm cash flows trigger downwards in order to perfectly meet the government cash flows trigger. This is exactly the effect that each of the optimal values for the contractual penalty produce and this effect is represented in Figure 4.2 by three arrows, one for each project dimension. It is visible that the greater the project dimension the greater the size of the corresponding arrow, hence reflecting that a higher contractual penalty is needed as we consider greater project dimensions. The private firm discriminatory boundary, formed by pairs $\{K_P^*; V_P^*\}$ is then situated between the private firm discriminatory boundary before considering the impact of the contractual penalty, formed by pairs $\{K_P^+; V_P^+\}$ and the government discriminatory boundary, formed by pairs $\{K_G^*; V_G^*\}$.[16]

Figure 4.2 also includes a specific point, which we designate as X. This point is situated where the private firm discriminatory boundary, after considering the impact of the optimal legal penalty (the one composed by pairs $\{K_P^*; V_P^*\}$), intersects the line for the government cash flows trigger on the second dimension, *i.e.*, $V_G^* = 247$. To this project dimension, the government cash flows trigger and private firm cash flows trigger are both equal to 247 and, to that cash flows trigger value, corresponds a private firm construction costs trigger of $K_P^* = 120$. Hence, considering the second dimension, given by $K_G^* = 200$, the private firm construction costs trigger, $K_P^* = 120$ when the comparative efficiency factor, $\gamma = 0.4$. This means that, if the comparative efficiency factor is equal or greater than 0.4, there is no need to include a legal penalty in the contract form because the efficiency factor - for a level of social costs

[16]The private firm discriminatory boundary, after considering the impact of the optimal legal penalty, would only match the government discriminatory boundary if the government assumed both entities to be equally efficient, *i.e.*, if $\gamma = 0$.

equal to 0.02 - is strong enough to move the private firm cash flows trigger downwards in order to perfectly meet the government cash flows trigger.

4.3.3 Sensitivity Analysis

4.3.3.1 The Impact of Variations in the Level of Social Costs

Table 4.3 includes a set of values for the level of social costs and their impact on the government cash flows trigger and on the optimal contractual penalty, assuming that the project dimension is given by $K_G^* = 200$.

Table 4.3: sensitivity analysis: the impact of variations in the level of social costs

(for: $\sigma_{V_G} = \sigma_{V_P} = 0.15$; $\sigma_{K_G} = \sigma_{K_P} = 0.10$; $\delta_{V_G} = \delta_{V_P} = 0.03$; $\delta_{K_G} = \delta_{K_P} = 0.03$; $\rho_G = \rho_P = 0$; $r = 0.05$; $\gamma = 0.2$)

K_G^*	K_P^*	s	V_G^*	$c*$	V_P^+	V_P^*
200	160	0.00	411	**0.0000**	329	329
200	**160**	**0.0075**	**329**	**0.0000**	**329**	**329**
200	160	0.01	308	**0.4732**	329	308
200	160	0.02	247	**1.9222**	329	247
200	160	0.03	206	**2.9235**	329	206
200	160	0.04	176	**3.6661**	329	176

Given a comparative efficiency factor, $\gamma = 0.2$, for values of $s \leq 0.0075$ there is no need to enforce a legal penalty because the private firm cash flows trigger is lower than (or equal to) the government cash flows trigger. For $s > 0.0075$, the higher the level of social costs the more below the government cash flows trigger is situated, which means that the difference between the private firm cash flows trigger and the government cash flows trigger becomes greater. Consequently, a stronger contractual penalty is needed in order to perfectly align the two triggers. This also implies that there is a specific level of social costs that makes both cash flows triggers have the same value, before considering the effect of any contractual penalty. Such level of social costs is, therefore, the threshold value below which the inclusion of a contractual penalty is not necessary. We can derive an analytical solution to determine this threshold value, which we designate as $\bar{s}$. Thus, $\bar{s}$ will be given by the following equation:[17]

$$\bar{s} \;=\; \frac{\gamma\delta_{V_P}}{1-\gamma} \tag{4.23}$$

As we are departing from equal values to the parameters that influence the value of β, both for the government and the private firm, we have reached a solution for $\bar{s}$, which is independent of β and merely depends on the private firm cash flows rate of return-shortfall, δ_{V_P} and on the level of the efficiency factor, γ. Considering the inputs used in our numerical example, $\bar{s} = 0.0075$. Therefore, this is the level of social costs that makes both cash flows triggers have the same value, as the results included in Table 4.3 reflect.

4.3.3.2 The Impact of Variations in the Level of the Efficiency Factor

Table 4.4 includes a set of different values for the comparative efficiency factor and the impact produced on the optimal contractual penalty, considering that the project dimension is given by $K_G^* = 200$.

[17] In the absence of social costs, considering greater levels for the comparative efficiency factor merely results in lower construction costs triggers for the private firm and, hence, lower cash flows triggers of the same magnitude. This implies that considering higher (lower) efficiency factors when social costs are absent only leads the private firm pair of triggers to be situated more backwards (forward) in the government discriminatory boundary. In these circumstances, the private firm discriminatory boundary and the government discriminatory boundary overlap each other, which means that no penalty is needed, regardless of the level of comparative efficiency.

Table 4.4: sensitivity analysis: the impact of variations in the level of the efficiency factor

(for: $\sigma_{V_G} = \sigma_{V_P}$ = 0.15; $\sigma_{K_G} = \sigma_{K_P}$ = 0.10; $\delta_{V_G} = \delta_{V_P}$= 0.03; $\delta_{K_G} = \delta_{K_P}$ = 0.03; $\rho_G = \rho_P$ = 0; r = 0.05; s = 0.02)

γ	K_G^*	K_P^*	V_G^*	$c*$	V_P^+	V_P^*
0.0	200	200	247	**3.9110**	411	247
0.1	200	180	247	**2.9087**	370	247
0.2	200	160	247	**1.9222**	329	247
0.3	200	140	247	**0.9523**	288	247
0.4	**200**	**120**	**247**	**0.0000**	**247**	**247**
0.5	200	100	247	**0.0000**	206	206

The results included in Table 4.4 demonstrate that higher efficiency factors lead to lower triggers for the private firm construction costs which, in turn, produce lower values for the corresponding cash flows triggers, V_P^+. Therefore, as the comparative efficiency factor, γ increases, the optimal contractual penalty, c^*, decreases, since both triggers are closer to each other, until a value is reached for the comparative efficiency which, by itself, is strong enough to align the two cash flows triggers. Hence, we conclude that, for given level of social costs, there is a value for the efficiency factor above which there is no need to include a contractual penalty in the contract form. In fact, we observed that Figure 4.2 includes the point X - whose coordinates are $\{120;247\}$. This point corresponds to this level of comparative efficiency and is situated where the private firm discriminatory boundary, before considering the effect of the contractual penalty, intersects the government discriminatory boundary, for the project dimension given by $K_G^* = 200$. When the efficiency factor is greater than 0.4, the private firm cash flows trigger is lower than the government cash flows trigger, meaning that there is no need to include a legal penalty in the contract because the government's purpose is already accomplished. In such cases, the optimal contractual penalty is zero, as the results included in Table 4.4 demonstrate.

In fact, the results included in Table 4.4 show that enforcing a legal penalty in the contract only becomes necessary for levels of comparative efficiency lower than 0.4, considering a level of social costs of 0.02. Similarly to what we did regarding the level of social costs, we can determine the threshold value for the efficiency factor below which the inclusion of a legal penalty in the contract form becomes necessary. Let $\bar{\gamma}$ denote the level of comparative efficiency that align both cash flows triggers: this is the level below which a legal penalty should be enforced. Being so, $\bar{\gamma}$ will be given by the following equation:[18]

$$\bar{\gamma} = \frac{s}{s+\delta_{V_P}} \tag{4.24}$$

4.3.3.3 The Combined Impact of Variations in the Level of Social Costs and in the Level of the Efficiency Factor

We will now demonstrate how variations in the level of social costs, combined with variations in the level of the comparative efficiency, affect the optimal value for the contractual penalty. As shown in Table 4.3, the greater the level of social costs the lower the government cash flows trigger. The government cash flows trigger is thus shifted downwards due to the existence of social costs, pressuring the government to invest sooner. However, we now have to confront the government cash flows trigger, affected by different levels of social costs, with the private firm cash flows trigger, affected by different levels of comparative efficiency. We emphasize that the optimal value for the contractual penalty is the one that moves the private firm cash flows trigger downwards with the purpose of perfectly meeting the government cash flows trigger, for any dimension the project may assume.

Table 4.5: sensitivity analysis: the combined impact of variations in the level of social costs and in the level of the efficiency factor

(for: $\sigma_{V_G} = \sigma_{V_P} = 0.15$; $\sigma_{K_G} = \sigma_{K_P} = 0.10$; $\delta_{V_G} = \delta_{V_P} = 0.03$; $\delta_{K_G} = \delta_{K_P} = 0.03$; $\rho_G = \rho_P = 0$; $r = 0.05$)

	$s = 0.01$	$s = \mathbf{0.02}$	$s = 0.03$	$s = 0.04$	$s = 0.05$
$\gamma = 0.1$	1.4311	**2.9087**	3.9359	4.7014	5.2996
$\gamma = \mathbf{0.2}$	**0.4734**	**1.9222**	**2.9235**	**3.6661**	**4.2443**
$\gamma = 0.3$	0.0000	**0.9523**	1.9290	2.6494	3.2078
$\gamma = 0.4$	0.000	**0.0000**	0.0000	1.6535	2.1926
$\gamma = 0.5$	0.000	**0.0000**	0.000	0.6802	1.2013

The values included in Table 4.5 are the optimal penalty values that result from considering

[18] Equation (4.24) is equivalent to equation (4.23), rewritten in terms of $\bar{\gamma}$.

different combinations for the level of social costs and for the level of the comparative efficiency. The results clearly illustrate what we have been arguing: the greater the level of social costs the higher the value of the optimal legal penalty and, the higher the efficiency factor, the lower the value of the optimal penalty. This means that a trade-off does exist between the two factors. The effect caused by a positive (negative) variation in one of the factors may be compensated by a negative (positive) variation in the other factor such that the optimal contractual penalty remains unchanged, as the difference between the private firm cash flows trigger and the government cash flows trigger remains the same.

The results included in Table 4.5 also show that when the comparative efficiency factor, $\gamma \geqslant 0.4$ and the level of social costs, $s \leqslant 0.03$, there is no need to include a legal penalty in the contract form because the combined effect of the two factors aligns the two cash flows triggers. The same happens when $\gamma = 0.3$ and $s = 0.01$. This is the reason why, in such cases, the optimal value for the contractual penalty included in Table 4.5 is zero.

4.3.3.4 The Impact of Variations in the Correlation Coefficients

We have been assuming that the correlation coefficients, ρ_G and ρ_P are zero and, hence, all calculations were made assuming the absence of correlation between the facility construction costs and the present value of cash flows to be generated from subsequent operations. Table 4.6 includes the results of the impact on the optimal contractual penalty caused by variations in both correlation coefficients, ρ_G and ρ_P, assuming $\gamma = 0.2$ and $s = 0.02$. We should stress that any change in ρ_G will cause the roots that are extracted from equation (4.8) to change and, thus, V_G^* will also change. The impact on V_P^+ is also presented. We set K_G^* to 200 and the conclusions are equivalent for any other level of K_G^* since the results included in Table 4.2 demonstrate the existence of proportionality between this trigger and the optimal level for the contractual penalty, c^*.[19]

[19]In order to confirm the existence of this proportionality, we set K_G^* to 100 and reached optimal penalty values, for all levels of ρ, half as great as the ones included in Table 4.6. These results confirm the existence of proportionality between K_G^* and c^*.

Table 4.6: the impact of variations in the correlation coefficients

(for: $\sigma_{V_G} = \sigma_{V_P} = 0.15$; $\sigma_{K_G} = \sigma_{K_P} = 0.10$; $\delta_{V_G} = \delta_{V_P} = 0.03$; $\delta_{K_G} = \delta_{K_P} = 0.03$; $r = 0.05$; $s = 0.02$; $\gamma = 0.2$)

$(\rho_G;\rho_P)$	**(-1.0;-1.0)**	**(-0.5;-0.5)**	**(0.0;0.0)**	**(0.5;0.5)**	**(1.0;1.0)**
K_G^*	200	200	200	200	200
V_G^*	320	284	247	205	147
V_P^+	427	379	329	273	196
V_P^*	320	284	247	205	147
c^*	**2.3213**	**2.1295**	**1.9222**	**1.6791**	**1.1967**

The results included in Table 4.6 show that one should expect that, as the correlation coefficients increase, from $(-1.0;-1.0)$ to $(1.0;1.0)$, the value of the optimal legal penalty would decrease, since stronger positive (or less negative) correlations lead to lower cash flows trigger values, both for the government and the private firm. In fact, it is expected that increasing positive (or less decreasing negative) correlations would result in lower cash flows triggers for both entities. Triggers will be closer to each other as positive (or less negative) correlations become greater and the differences between the triggers become smaller. And, since the difference between the triggers becomes smaller, then a lower value for the contractual penalty will be sufficient to perfectly align the triggers. On the contrary, higher negative correlations (or lower positive correlations) will, expectedly, result in greater differences between the private firm cash flows trigger - in the event that no penalty is enforced - and the government cash flows trigger, meaning that, in these circumstances, a greater value for the legal penalty will be needed in order to move the firm cash flows trigger downwards and perfectly align it with the government cash flows trigger.

4.3.3.5 The Impact of Variations in the Standard Deviations

Table 4.7 illustrates how variations in both standard deviations affect the optimal contractual penalty value. We consider variations in $(\sigma_K;\sigma_V)$ ranging from $(0.05;0.05)$ to $(0.30;0.30)$. We should stress that any change considered in the volatility parameters will cause the roots that are extracted from equation (4.8) to change and, thus, V_G^* will also change.

Table 4.7: the impact of variations in the standard deviations

(for: K_G^*= 200; $\delta_{V_G} = \delta_{V_P}$= 0.03; $\delta_{K_G} = \delta_{K_P}$ = 0.03;$\rho_G = \rho_P$ = 0; r = 0.05; s = 0.02; γ = 0.2)

$(\sigma_k;\sigma_V)$	**(0.05; 0.05)**	**(0.10; 0.10)**	**(0.15; 0.15)**	**(0.20; 0.20)**	**(0.25; 0.25)**	**(0.30; 0.30)**
V_G^*	160	212	278	360	459	575
V_P^+	213	283	371	480	611	767
c^*	**1.7889**	**2.0044**	**2.2350**	**2.4579**	**2.6631**	**2.8454**
V_P^*	160	212	278	360	459	575

The results included in Table 4.7 show that higher volatility levels lead to higher values for both the government cash flows trigger and the private firm cash flows trigger, as expected. However, the difference between the private firm cash flows trigger before considering the effect of the contractual penalty, V_P^+ and the government cash flows trigger, V_G^* is greater as the volatility levels increase. This increasing difference between the two cash flows triggers is explained by the combined effect that the efficiency factor and the level of social costs produce in the private firm cash flows trigger and in the government cash flows trigger, respectively. Assuming that $\gamma = 0.2$ and $s = 0.02$, the effect produced by the level of social costs in moving the government cash flows trigger downwards is stronger than the effect caused by the efficiency factor in placing the private firm cash flows trigger more below. This greater difference between the two triggers results in higher values for c^*, since the effect produced by the legal penalty needs to be stronger in order to move the private firm cash flows trigger downwards and perfectly meet the government cash flows trigger.[20]

4.4 The Effects, to the Government, of a Including a Non-Optimal Value for the Legal Penalty in the Contract

The model we have described proposes a method to determine the optimal level for the contractual penalty, from the government or other public entity perspective. Based on its own

[20] We determined the government cash flows triggers and the private firm cash flows triggers, first not considering the effect caused by the level of the efficiency factor and, secondly not considering the effect caused by the level of social costs. The difference between the two cash flows triggers is greater in the first case, which demonstrates that the effect produced by the level of social costs is stronger, for these combinations of standard deviations.

estimates about the facility construction costs and the value of cash flows to be generated from running the subsequent operations, and estimating the level of social costs and the comparative efficiency factor, the government is able to reach the value for the optimal contractual penalty "per unit" of the private firm's expected construction costs. This optimal contractual penalty moves the private firm cash flows trigger downwards to perfectly meet the government cash flows trigger and, thus, the two cash flows triggers have the same value. However, since we are assuming the government construction costs to be stochastic, one may legitimately ask which value for these costs should the government consider for the purpose of determining the optimal penalty to be included in the contract form, in the day the optimal contractual penalty needs to be determined. For the sake of simplicity, we assume that the definition of the contractual penalty by the government and the bidding process occur in the same moment of time. Therefore, and since any rational agent makes decisions based on the best available information he or she has in the specific moment the decisions need to be taken, it is reasonable to conclude that the estimated value for the construction costs the government has computed in that same moment will be the value the government should use for the purpose of determining the optimal contractual penalty. However, since the government construction costs behave stochastically, their value might be different the day after the day the value for the contractual penalty was determined. If this occurs, the value of the legal penalty determined and included in the contract form the day before will no longer be optimal. However, since the government needs to make a decision, the optimal value for the contractual penalty, on that day, is determined by the government, based on the expected construction costs estimated on that same day, and included in the contract form. Bidders prepare their proposals taking into account the presence of this contractual penalty and establish their bid prices accordingly. One of the bidders is selected and invited to sign the contract and, consequently, to invest in executing the project facility and run the subsequent operations.

Yet, one may consider that the value of the legal penalty included in the contract form is not optimal - not only due to the fact the government construction costs behave stochastically, as we just mentioned - but also because the government might have inaccurately estimated the selected bidder's comparative efficiency. In fact, it is possible that the government has determined the value of the contractual penalty assuming a comparative efficiency factor which is different from the selected bidder's real comparative efficiency. Being so, we should examine the effects that overestimating or underestimating the efficiency factor will produce, from the government's perspective. For this purpose, we need to present the value of the selected bidder investment opportunity.

Let $F_B(V_B, K_B)$ denote the value of the selected bidder option to invest in a project where a legal penalty will be enforced in the case the selected firm does not implement the project immediately. V_B designates the present value of the cash flows the selected bidder will generate by running the operations and K_B designates the selected bidder facility construction costs. Considering the same assumptions we have considered when we derived $F_P(V_P, K_P)$, *i.e.*, the government expectation about the private firm decision to invest, and applying the same procedures, we reach the boundary that separates the selected bidder's waiting and investing regions.[21]

Let c' denote the value of the contractual penalty effectively included in the contract. Being so, the value of the selected bidder investment opportunity is given as follows:

$$F_B(V_B, K_B) = \begin{cases} (V_B^* - K_B^* + \frac{c'}{r}) \left(\frac{V_B}{V_B^*}\right)^{\beta^+} \left(\frac{K_B}{K_B^*}\right)^{\eta^-} - \frac{c'}{r}; if & \{K_B, V_B\} < \{K_B^*, V_B^*\} \\ V_B - K_B \qquad ; if & \{K_B, V_B\} \geq \{K_B^*, V_B^*\} \end{cases} \tag{4.25}$$

We will discuss each of the two non-optimal scenarios separately. First, we present the scenario where the government overestimated the selected bidder comparative efficiency and, later, the scenario where the selected bidder efficiency factor is greater than the government expected. Again, since the efficiency factor plays a crucial role in determining the optimal value for the legal penalty, any difference that may exist between the efficiency factor estimated by the government and the selected bidder's real comparative efficiency will necessarily mean that the value of the legal penalty included in the contract is not optimal. As we will demonstrate, overestimating (underestimating) the selected bidder comparative efficiency factor will lead to the inclusion of a below-optimal (above-optimal) value for the legal penalty in the contract form.

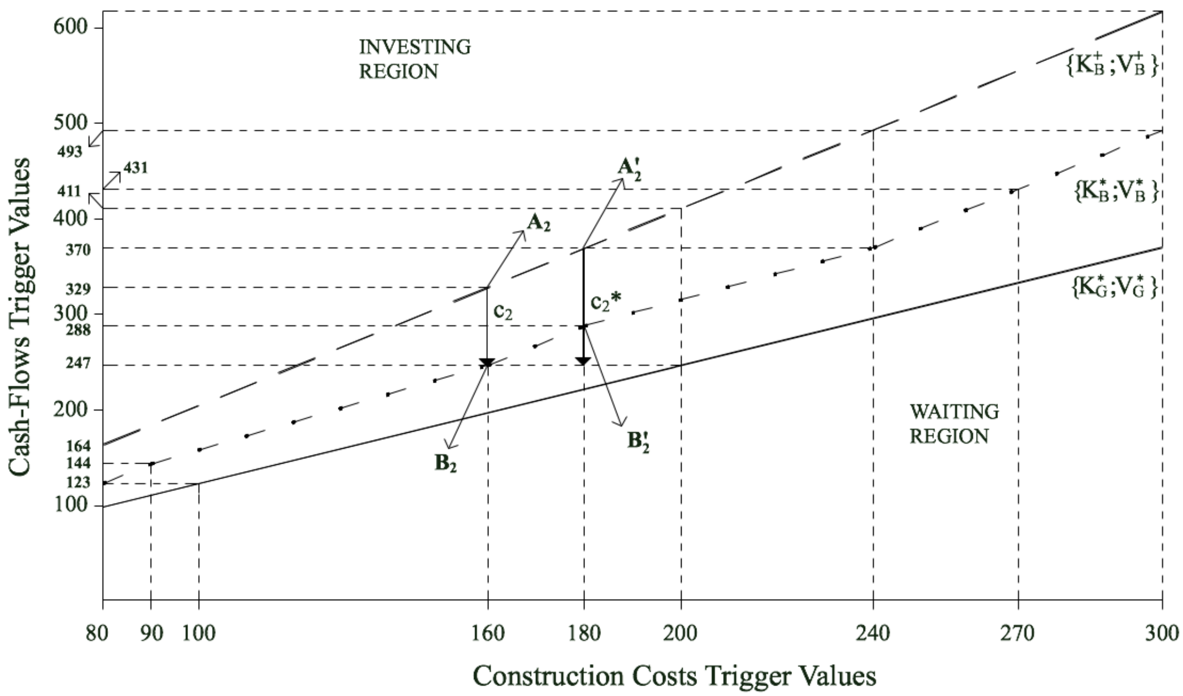
4.4.1 The Effects of Including a Below-Optimal Penalty Value in the Contract

Figure 4.3 illustrates the effects of overestimating the selected bidder comparative efficiency. We assume that the government estimated the selected bidder comparative efficiency as being $\gamma = 0.2$, and we will consider that the selected bidder comparative efficiency is, in fact, $\gamma = 0.1$. We will also consider that the project dimension is given by $K_G^* = 200$. Figure 4.3

[21] Please refer to Appendix A for further details.

shows the effects caused by overestimating the selected bidder comparative efficiency and the corresponding impact on the optimal contractual penalty.

Figure 4.3: the impact of overestimating the comparative efficiency on the optimal contractual penalty



In Figure 4.3, point A_2 has the coordinates that correspond to the selected bidder pair of triggers before considering the effect of the contractual penalty, assuming the efficiency factor estimated by the government, *i.e,* $\{K_B^+;V_B^+\} = \{160;329\}$, whereas point B_2 graphically represents the selected bidder pair of triggers, $\{K_B^*;V_B^*\} = \{160;247\}$ after considering the impact of the legal penalty enforced in the contract, $c_2 = 1.9222$. In these circumstances, the government would expect the value of legal penalty included in the contract form, c_2 to be sufficiently strong in moving the selected bidder cash flows trigger downwards, from $V_B^+ = 329$ to $V_B^* = 247$ and, thus, perfectly meet the government cash flows trigger, $V_G^* = 247$. By overestimating the efficiency factor, the government has not succeeded in reaching this purpose. The government was expecting the selected bidder to trigger the investment in $\{K_B^*;V_B^*\} = \{160;247\}$, represented by point B_2, as the result of enforcing the contractual penalty, $c_2 = 1.9222$. If no penalty was enforced, and given the selected bidder's real comparative efficiency, he or she would trigger the investment in point A_2' , whose coordinates are $\{K_B^+; V_B^+\} = \{180; 370\}$. By enforcing the contractual penalty $c_2 = 1.9222$, when the selected bidder comparative efficiency is $\gamma = 0.1$, the government is leading the selected bidder to trigger the investment in point $B_2^{'}$, whose coordinates are $\{K_B^*; V_B^*\} = \{180; 288\}$. In fact, the selected bidder construction costs trigger, considering his or her real comparative efficiency, is $K_B^* = 180$, and the corresponding cash flows trigger is $V_B^* = 288$.[22] Thus, by imposing the contractual penalty, $c_2 = 1.9222$, the government has led the selected bidder cash flows trigger to move from $V_B^+ = 370$ to $V_B^* = 288$. Hence, the government's purpose of moving the selected firm cash flows trigger downwards to perfectly meet the government cash flows trigger has not been accomplished, because the selected bidder cash flows trigger is higher than the government cash flows trigger. This means that a greater penalty value should be enforced in order to move further downwards the selected bidder cash flows trigger. In Figure 4.3, the optimal contractual penalty is given by c_2^* and is clearly visible that the line segment that reflects the impact of the optimal contractual penalty, c_2^* is bigger than the line segment that reflects the impact caused by the value of the legal penalty actually included in the contract, $c_2 = 1.9222$. This fact graphically illustrates that overestimating the comparative efficiency factor leads to the definition of a below-optimal value for the contractual penalty.

Table 4.8 includes a set of representative values aiming to demonstrate the effects, to government, of including a below-optimal contractual penalty in the contract form. We assume that the project dimension is given by $K_G^* = 200$ and we also consider that the pair of observed values in the market, at time t, for the selected bidder construction costs and the corresponding cash flows is $\{K_{B_t}; V_{B_t}\} = \{220; 180\}$. We are therefore considering the case where, at time t, the selected bidder decision to invest is situated in the waiting region.

Table 4.8: the effects, to the government, of overestimating the comparative efficiency

(for: $\sigma_{V_G} = \sigma_{V_P} = 0.15$; $\sigma_{K_G} = \sigma_{K_P} = 0.10$; $\delta_{V_G} = \delta_{V_P} = 0.03$; $\delta_{K_G} = \delta_{K_P} = 0.03$; $\rho_G = \rho_P = 0$; $r = 0.05$; $s = 0.02$)

	c_2	V_G^*	K_B^+	V_B^+	K_B^*	V_B^*	K_{B_t}	V_{B_t}	$F_{B_t}(V_{B_t}, K_{B_t})$
$\gamma = 0.1$	1.9222	247	180	370	180	288	220	180	45.30
$\gamma = 0.2$	1.9222	247	160	329	160	247	220	180	44.72

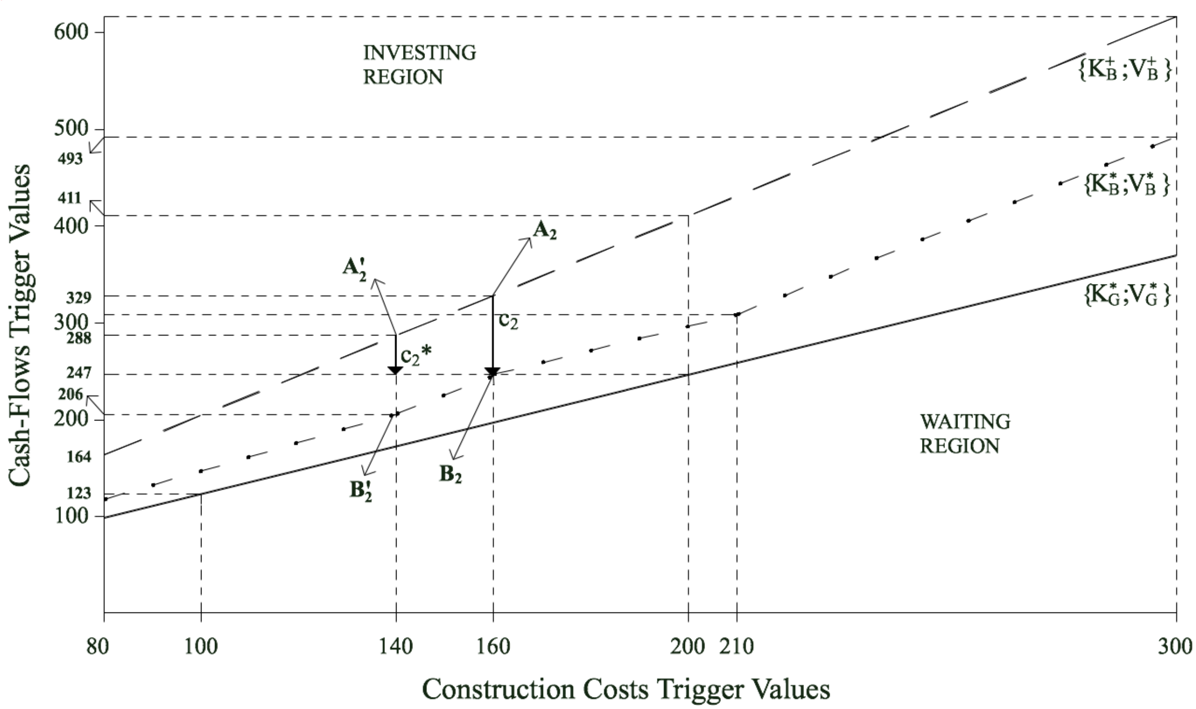
[22]For a given value of the selected bidder construction costs trigger, the corresponding cash flows trigger is determined using a system of simultaneous equations similar to the one composed by equations (4.20), (4.21) and (4.22), *i.e.*, the ones we used to derive the government expectation about the private firm investment opportunity. Please refer to Appendix A for further details.

We have concluded that the government should have defined a greater value for the contractual penalty than the one actually enforced in the contract. The optimal contractual penalty is $c_2^* = 2.9087$: this is the value for the legal penalty that moves the selected bidder cash flows trigger, $V_B^+ = 370$ downwards to perfectly meet the government cash flows trigger, $V_G^* = 247$. The optimal contractual penalty, $c^* = 2.9087$ assumes a greater value than the one included in the contract, *i.e.*, $c_2 = 1.9222$. Hence, overestimating the comparative efficiency factor results in the definition of a below-optimal level for the contractual penalty, leading the selected bidder cash flows trigger to be placed above the government cash flows trigger. In fact, assuming $K_G^* = 200$, the selected bidder construction costs trigger is $K_B^* = 180$ for $\gamma = 0.1$ and its cash flows trigger, $V_B^* = 288$, considering the value of the penalty included in the contract form, $c_2 = 1.9222$. Since the selected bidder will have a higher cash flows trigger than the government predicted, the value of social costs will be greater than the value the government considers to be acceptable, which means that the population will have to bear a higher value of social costs. Yet, the fact that the contractual penalty is below its optimal level results in a more valuable option to invest for the selected bidder "vis-a-vis" with the scenario where the contractual penalty written down in the contract is optimal. The information included in Table 4.8 confirms such. Assuming that the pair of observed values, at time t, $\{K_{B_t}; V_{B_t}\} = \{220; 180\}$, the value of the selected bidder investment opportunity, $F_B(K_B, V_B)$, at time t equals 45.30, whereas $F_B(K_B, V_B)$, at time t, in the scenario where the legal penalty written down in the contract is optimal, equals 44.72. This numerically demonstrates that the value of the option to invest is more valuable in the presence of a below-optimal contractual penalty. Since we believe that rational bidders will offer higher prices as a direct result of holding a more valuable option to invest, the government would therefore receive a higher price. However, the increase in the expected value of social costs that results from delaying the project implementation is not compensated by the extra-income the government would most likely receive. In fact, the government would prefer to receive a lower price and ensure that the value of of social costs considered to be tolerable is not exceeded. Hence, the scenario where a below-optimal contractual penalty value is considered will produce effects that the government would prefer to prevent.

4.4.2 The Effects of Including an Above-Optimal Penalty Value in the Contract

Figure 4.4 illustrates the impact of underestimating the selected bidder comparative efficiency on the optimal contractual penalty. We assume, in this scenario, that the selected bidder comparative efficiency is, in fact, $\gamma = 0.3$. As in the previous scenario, we consider that the project dimension is given by $K_G^* = 200$.

Figure 4.4: the impact of underestimating the comparative efficiency on the optimal contractual penalty



In Figure 4.4 above, the point A_2, with coordinates $\{K_B^+; V_B^+\} = \{160; 329\}$, represents the selected bidder pair of triggers before considering the effect of the contractual penalty and assuming the efficiency factor estimated by the government ($\gamma = 0.2$). The coordinates of point B_2 represent the selected bidder pair of triggers, $\{K_B^*; V_B^*\} = \{160; 247\}$, after considering the impact of the legal penalty included in the contract, $c_2 = 1.9222$. By including this penalty value in the contract, the government expected the contractual penalty $c_2 = 1.9222$ to be strong enough in moving the selected bidder cash flows trigger downwards, from $V_B^+ = 329$ to $V_B^* = 247$ and, thus, to perfectly meet the government cash flows trigger, $V_G^* = 247$. So, the government expected the selected bidder to trigger the investment in point B_2, with coordinates $\{K_B^*; V_B^*\} = \{160; 247\}$. However, since we are considering that the government has underestimated the selected bidder efficiency factor - which is $\gamma = 0.3$ and not $\gamma = 0.2$ - then the selected bidder construction costs trigger is $K_B^* = 140$ and his or her corresponding cash flows trigger, $V_B^* = 206$. This pair of triggers is represented in Figure 4.4 by point $B_2^{'}$, with coordinates $\{K_B^*; V_B^*\} = \{140; 206\}$. In Figure 4.4, is visible that the value of the legal penalty enforced in the contract, c_2 is too high: the impact in moving the selected bidder cash flows trigger from $V_B^+ = 329$ to $V_G^* = 247$ was too strong and shifted the trigger further downwards to $V_B^* = 206$. We thus conclude that the optimal value for the contractual penalty would have to be smaller in order to perfectly align the two cash flows triggers. For this purpose to be accomplished, the optimal value for the contractual penalty would have to be $c_2^* = 0.9523$, a lower value than the one actually included in the contract, $c_2 = 1.9222$.

Table 4.9 includes a set of representative values aiming to demonstrate the effects, to government, of including an above-optimal contractual penalty in the contract form. As in the previous scenario, we consider that the project dimension is given by $K_G^* = 200$ and also that the pair of observed values in the market, at time t for the selected bidder construction costs and cash flows is $\{K_{B_t}; V_{B_t}\} = \{220; 180\}$, which means that the selected bidder decision to invest, at time t, is situated in the waiting region.

Table 4.9: the effects, to the government, of underestimating the comparative efficiency

(for: $\sigma_{V_G} = \sigma_{V_P} = 0.15$; $\sigma_{K_G} = \sigma_{K_P} = 0.10$; $\delta_{V_G} = \delta_{V_P} = 0.03$; $\delta_{K_G} = \delta_{K_P} = 0.03$; $\rho_G = \rho_P = 0$; $r = 0.05$; $s = 0.02$)

	c_2	V_G^*	K_B^+	V_B^+	K_B^*	V_B^*	K_{B_t}	V_{B_t}	$F_{B_t}(V_{B_t}, K_{B_t})$
$\gamma = 0.2$	1.9222	247	160	329	160	247	220	180	44.72
$\gamma = 0.3$	1.9222	247	140	288	140	206	220	180	43.42

Since the selected bidder is more efficient than the government predicted (in our example, the selected bidder's real comparative efficiency is $\gamma = 0.3$), we concluded that the government should have defined a contractual penalty, $c_2^* = 0.9523$, thus smaller than the one actually enforced in the contract, $c_2 = 1.9222$. Consequently, the selected bidder cash flows trigger assumes a lower value, $V_B^* = 206$, leading to a reduction in the level of social costs. We recognize that a reduction in the value of social costs results in a benefit for the population and argue that the government will welcome any benefit for the population that may rise, provided that such benefit does not entail a cost.[23] However, this benefit for the population involves a cost to the government. Since the penalty value included in the contract form is lower than the optimal value, we expect bidders to act rationally and offer a lower bid price to the government as a consequence of holding a less valuable investment opportunity, when

[23] This is exactly the case where the comparative efficiency factor is strong enough to place the selected bidder cash flows trigger below the government cash flows trigger, without the need of imposing a contractual penalty.

compared with the case where the value of the legal penalty included in the contract form is optimal. In fact, the selected bidder option to invest, at time t, $F_{B_t}(V_{B_t}, K_{B_t})$ equals 43.42, whereas the value of the same option in the case the government has estimated correctly the comparative efficiency is, at time t, equal to 44.72. We recognize that one might consider that the decrease in the bid price the selected bidder will probably offer actually compensates the benefit that derives from the reduction in the value of social costs. However, we argue that the government will prefer to tolerate a higher value of social costs until the value determined has being acceptable in the first place and, thus, not expose itself to the possibility of receiving a lower bid price than the one the government would receive if the value of the contractual penalty was, in fact, optimal.

4.5 Concluding Remarks

The suggested model is based on a contractual framework, in the context of a BOT project, where the government or other public entity grants leeway to the selected bidder regarding the timing for project implementation. However, this flexibility may entail a cost since a contractual penalty is enforced in the event the selected firm does not implement the project immediately. Aiming to determine the optimal value for this contractual penalty, we applied a two-factor uncertainty approach where both the facility construction costs and the present value of cash flows to be generated by subsequent operations follow geometric Brownian motions that are possibly correlated.

We have assumed that the government is less efficient than the private firm in constructing the project facility and we have also considered the existence of social costs, *i.e.*, the costs that correspond to the loss of social welfare occurring from the moment the project should have been implemented and services being provided to the users and the moment the project is actually completed.

In order to determine the optimal level for the contractual penalty, we derived both the government decision to invest and the government expectation about the private firm decision to invest. As the latter does not support homogeneity of degree one in the corresponding boundary conditions, due to the inclusion of the legal penalty in the value of the investment opportunity, we followed Adkins and Paxson (2011) approach and constructed a set of three simultaneous equations, which enabled us to define the private firm discriminatory boundary, *i.e.*, the boundary that separates the private firm's waiting region from the private firm's investing region. We then applied again the same system of equations and reached the optimal value for the contractual penalty "per unit" of cost, since proportionality is present between the optimal value for the contractual penalty and the expected amount of construction costs.

Sensitivity analysis performed demonstrated that the greater the level of social costs the sooner one expects the government would invest, since the consideration of higher levels of social costs result in lower values for the government cash flows trigger. The analysis also revealed that there is a level for the comparative efficiency above which there is no need to include a penalty in the contract because such level of comparative efficiency - for the level of social costs considered - naturally moves the private firm cash flows trigger downwards to perfectly meet the government cash flows trigger. Similarly, there is a level of social costs below which imposing a contractual penalty is not justifiable, for a given level of comparative efficiency. Sensitivity analysis also led us to conclude that changes in the correlation coefficients have a strong expected impact on the optimal contractual penalty. As both coefficients assume greater values, the value of the optimal penalty decreases since one expects stronger correlations to lead to lower cash flows triggers, both for the government and the private firm. We also assessed the impact of variations in both standard deviations on the optimal contractual penalty. The results demonstrate that higher volatility levels result in higher values for both the government cash flows trigger and the private firm cash flows trigger, as expected. However, the difference between the private firm cash flows trigger before considering the effect of the contractual penalty and the government cash flows trigger is greater as the volatility levels increase. This increasing difference between the two cash flows triggers is explained by the combined effect that the efficiency factor and the level of social costs produce in the private firm cash flows trigger and in the government cash flows trigger, respectively. Indeed, the effect produced by the level of social costs in moving the government cash flows trigger downwards is stronger than the effect produced by the efficiency factor in placing the private firm cash flows trigger more below. This greater difference between the two triggers results in the definition of higher optimal contractual penalty values.

Finally, we examined the effects that overestimating or underestimating the comparative efficiency factor produces in the value of the optimal contractual penalty. We concluded that overestimating the efficiency factor results in the definition of a below-optimal value for the contractual penalty, and that underestimating this same factor will lead to the inclusion of an-above optimal value for the legal penalty in the contract form. Being so, since overestimating (underestimating) the value for the comparative efficiency factor will result in the inclusion of a below-optimal (above-optimal) value for the legal penalty in the contract form, then the selected bidder cash flows trigger will be greater (smaller) than the government cash flows trigger. Since we defined the optimal contractual penalty as the one that ensures that the selected firm has the same value for his or her cash flows trigger than the government would have if the government decided to undertake the project, we then concluded that the government's purpose will never be accomplished if the comparative efficiency factor is not estimated with full accuracy.

Chapter 5

Conclusion

Three theoretical models were proposed in this dissertation. The first two models, presented in Chapters II and III, are support decision models intended to be used by managers of construction companies whose business consists in performing construction projects awarded through adequate bidding competitions. The third model, suggested in Chapter IV, is intended to be applied by governments or other public entities in the context of Build-Own-Transfer (BOT) projects.

The model presented in Chapter II aims to contribute to the optimal mark-up bid debate using the real options approach. The option to sign the contract and perform the project constitutes a real option since construction costs behave stochastically from the moment the bid price is established and the moment the client selects one of the bidders and, also, because the selected bidder has flexibility regarding the decision of whether to sign the contract and, consequently, invest in executing the project. The numerical solution we suggested, consisting of a maximization problem, determines that to the highest value of the option to sign the contract weighted by the probability of winning the bid, corresponds a specific price and, therefore, a specific mark-up bid. Considering the characteristics of the real option we have identified, this price is the optimal price and, hence, the price construction managers should include in their bid proposals.

Sensitivity analysis performed to the option volatility level and the option 'time to expiration' parameter revealed that the maximum value of the option is higher when greater levels both for the volatility and the 'time to expiration' parameters are considered. However, to this higher maximum option value corresponds a lower optimal bid price since construction managers will include a lower mark-up bid as a consequence of holding a more valuable option to invest. A sensitivity analysis was also performed to the project dimension, given by the expected value of construction costs. Results demonstrated that no scale-effect is present in the model since the consideration of greater construction costs values produces no impact on the optimal mark-up bid.

Sensitivity analysis was also performed to the two parameters included in the proposed winning function. The results clearly showed that the model's outcome is highly sensitive to variations in both them. However, the optimal bid price is particularly sensitive to variations in parameter 'n' (the one responsible for shaping the winning function's concavity and convexity) when this parameter assumes very low levels. Therefore, construction managers should be aware that variations in this parameter when very low levels are considered will produce a strong impact on the optimal bid price.

An extension to the model was also proposed in order to accommodate the existence of penalty costs, borne by the selected bidder in the event he or she declines the invitation to sign the contract. The numerical solution was adapted with the purpose of considering these type of costs and, according to the inputs considered in the numerical example, the optimal price is greater when penalty costs are present. Based on the same inputs, we also concluded that the higher the level of penalty costs the higher the optimal price.

The model suggested in Chapter III addressed the impact that volume uncertainty may produce on the project value and on the optimal bid price. This type of uncertainty assumes a critical importance since, frequently, construction managers do not know, at the bid preparation stage, the exact amount of work that will be performed during the project's life cycle and, hence, the expected final profit the project will generate. To assess the impact of volume uncertainty on the project value, we defined a discrete-time stochastic variable and designated it as "additional value". Additional value is the value that is hidden in the the most uncertain portions of the project and is defined as the one that does not result from solely performing the tasks specified in the initial contract.

To capture and quantify this hidden-value, construction companies need to invest. Initially, by merely using the skills of his or her own experienced staff, managers are able to define a high-value estimate and a low-value estimate for the additional profit and to stipulate a probability of occurrence to each of the estimates. Applying the maximization procedure suggested in Chapter II, we concluded that managers will produce a more competitive bid price even if no incremental investment is undertaken, provided that some hidden-value is captured and quantified during the bid preparation stage. However, in order to resolve the uncertainty concerning which of the two estimates will become the true value for the expected additional profit, contractors often need to invest in human capital and technology and, thus, hire specialized firms and skilled consultants. The model's outcome is the threshold value for this incremental investment in human capital and technology. A decision rule was reached: managers should hire external services with the purpose of resolving the uncertainty regarding which of the two estimates previously defined is the true value, as long as the cost of this incremental investment in human capital and technology does not exceed the predetermined threshold value. Hence, we concluded that any amount spent in hiring external services, which is lower than the threshold value, will lead to an increase in the value of the project. On the contrary, if the amount invested is greater than the predetermined threshold value, then the project value will decrease.

The model also enables managers to establish the optimal bidding price if (i) no incremental investment is undertaken; (ii) if the incremental investment in human capital and technology reveals that the true value for the additional profit equals the high-estimate and (iii) if this investment reveals that the true value for the additional profit equals the low-estimate.

Sensitivity analysis showed that the incremental investment threshold value responds linearly to variations in the project dimension, which means that no scale-effect is present. Sensitivity analysis also showed that, the closer to 50% is the probability of occurrence associated with the estimates, the greater the threshold value is, since undertaking the incremental investment assumes a higher importance in response to the presence of higher levels of uncertainty regarding which of the two estimates will become the true value. Finally, sensitivity analysis performed to the difference between the two estimates revealed that, the greater the difference between the two estimates defined for the additional value, the greater the level of uncertainty regarding which of the two estimates will become the true value. Therefore, the incremental investment assumes a greater importance in resolving this uncertainty and this greater importance is reflected in a greater incremental investment threshold value. On the contrary, if the difference between the estimates is smaller, the level of uncertainty is lower, which means that the incremental investment assumes a lower importance in eliminating this uncertainty and, hence, the threshold for the incremental investment assumes a smaller value.

In Chapter IV, we turned our attention to BOT projects and suggested a model, based on a contractual framework where the government or other public entity does not impose any obligation regarding the timing for project implementation. However, the presence of this flexibility may entail a cost to the selected bidder since a contractual penalty is enforced in the case he or she does not initiate the project immediately. In order to determine the optimal value for the contractual penalty, we applied a two-factor uncertainty approach where both the facility construction costs and the present value of cash flows to be generated by running the subsequent activities follow geometric Brownian motions that are possibly correlated.

We have assumed that the government is less efficient than the private firm in executing the project facility and have also considered the existence of social costs, *i.e.*, the costs that correspond to the loss of social welfare occurring from the moment the project should have been implemented and services provided to the users and the moment the project is actually completed.

In order to determine the optimal level for the contractual penalty, we derived both the government decision to invest and the government expectation about the private firm decision to invest. As the latter does not support homogeneity of degree one in the corresponding boundary conditions, due to the inclusion of the legal penalty in the value of the option to invest, we followed Adkins and Paxson (2011) quasi-analytical approach and defined a set of three simultaneous equations. This set of equations enabled us to determine the private firm discriminatory boundary, separating the private firm's waiting and investing regions and to reach the optimal level for the contractual penalty. Moreover, since proportionality is present between the optimal contractual penalty value, in absolute terms, and the private firm expected construction costs, we determined the optimal value for the contractual penalty "per unit" of the private firm expected construction costs.

Sensitivity analysis performed demonstrated that the greater the level of social costs the sooner one expects the government to invest. In fact, the consideration of higher levels of social costs result in lower values for the government cash flows trigger. Sensitivity analysis also showed that there is a level for the comparative efficiency above which there is no need to include a penalty in the contract because such level of comparative efficiency, for a given level of social costs, naturally shifts the private firm cash flows trigger downwards to perfectly meet the government cash flows trigger. In similar terms, a level of social costs does exist above which imposing a contractual penalty becomes necessary, for a given level of comparative efficiency. We then presented the analytical solution that determines these two threshold values.

Sensitivity analysis also revealed that variations in both correlation coefficients have a strong expected impact on the optimal contractual penalty. We also examined the impact produced by variations in both standard deviations on the optimal contractual penalty. We concluded that the consideration of higher volatility levels result in higher cash flows trigger values both for the government and the private firm. Moreover, we concluded that, as the level of volatility increases, the difference between the private firm cash flows trigger and the government cash flows trigger becomes greater. This fact is explained by the stronger impact produced by the level of social costs in the government cash flows trigger when compared with the impact produced by the comparative efficiency factor in the private firm cash flows trigger. Consequently, higher contractual penalties are needed to align the two cash flows triggers.

Finally, we studied the effects that overestimating or underestimating the comparative efficiency factor produces in the optimal contractual penalty. We concluded that overestimating (underestimating) the efficiency factor results in the definition of a below-optimal (above-optimal) value for the contractual penalty. Being so, since overestimating (underestimating) the value for the comparative efficiency factor will lead to the inclusion of a below-optimal (above-optimal) value for the legal penalty in the contract form, then the selected bidder cash flows trigger will be higher (lower) than the government cash flows trigger. Since the optimal contractual penalty is the one that ensures that the selected firm has the same value for his or her cash flows trigger the government would have if the government decided to undertake the project, we concluded that the government's purpose will never be accomplished if the comparative efficiency factor is not estimated with full precision.

Appendix A

The Selected Bidder Decision to Invest

Let $F_B(K_B, V_B)$ denote the value of the selected bidder investment opportunity, where K_B designates the facility construction costs and V_B the value of the cash flows to be generated by operating the subsequent activities. Both stochastic variables follow geometric Brownian motions that are possibly correlated:

$$dV_B = \alpha_{V_B} V_B dt + \sigma_{V_B} V_B dz_{V_B} \tag{A.1}$$

$$dK_B = \alpha_{K_B} K_B dt + \sigma_{K_B} K_B dz_{K_B} \tag{A.2}$$

$$E(dz_{V_B}, dz_{K_B}) = \rho_B dt \tag{A.3}$$

where α_{V_B} and α_{K_B} are the drift parameters, dt is the time interval, σ_{V_B} and σ_{K_B} the standard deviations for each of the variables, dz_{V_B} and dz_{K_B} the increments of standard Wiener processes for V_B and K_B and ρ_B is the correlation coefficient between V_B and K_B.

Under risk-neutrality, $F_B(V_B, K_B)$ must satisfy the following partial differential equation:

$$\frac{1}{2}\sigma_{V_B}^2 V_B^2 \frac{\partial^2 F_B}{\partial V_B^2} + \frac{1}{2}\sigma_{K_B}^2 K_B^2 \frac{\partial^2 F_B}{\partial K_B^2} + \rho_B \sigma_{V_B} \sigma_{K_B} V_B K_B \frac{\partial^2 F_B}{\partial V_B \partial K_B} + (r-\delta_{V_B})V_B \frac{\partial F_B}{\partial V_B} + (r-\delta_{K_B})K_B \frac{\partial F_B}{\partial K_B} - rF_B - c' = 0 \quad (A.4)$$

where r is the risk-free interest rate, c' is the value of the legal penalty enforced in the contract, δ_{V_B} and δ_{K_B} are the rates of return-shortfall for V_B and K_B, respectively. δ_{V_B} and δ_{K_B} are given by the following equations:

$$\delta_{V_B} = r - \alpha_{V_B} \quad (A.5)$$

$$\delta_{K_B} = r - \alpha_{K_B} \quad (A.6)$$

The partial differential equation $(A.4)$ has a non-homogeneous part, c', being the rest of the equation homogeneous. The general solution for the homogeneous part of the partial equation , $F_B^H(V_B, K_B)$ is given by:

$$F_B^H(V_B, K_B) = A_9 V_B^{\beta^+} K_B^{\eta^+} + A_{10} V_B^{\beta^+} K_B^{\eta^-} + A_{11} V_B^{\beta^-} K_B^{\eta^+} + A_{12} V_B^{\beta^-} K_B^{\eta^-} \quad (A.7)$$

where A_9, A_{10}, A_{11} and A_{12} are constants that need to be determined. β^+, β^-, η^+ and η^- are the four roots of an elliptical equation, which is the two-factor equivalent of the one-factor stochastic model quadratic equation presented in Dixit and Pindyck (1994). The elliptical equation is as follows:

$$Q(\beta, \eta) = \frac{1}{2}\sigma_{V_B}^2 \beta(\beta-1) + \frac{1}{2}\sigma_{K_B}^2 \eta(\eta-1) + \rho_B \sigma_{V_B} \sigma_{K_B} \beta\eta + (r-\delta_{V_B})\beta + (r-\delta_{K_B})\eta - r = 0 \quad (A.8)$$

The function $Q(\beta, \eta) = 0$ defines an ellipse.[1] Equation $(A.8)$ has four quadrants, to which of them corresponds a pair of the four roots we mentioned, *i.e.*, the four points where the

[1] Please refer to Adkins and Paxson (2011) for further details.

elliptic function meets the axes. If we consider the absorbing barriers $F_B(0,0)$, $F_B(0,K_B)$, then $A_3 = A_4 = 0$. This condition holds because the option to invest is worthless if the present value of the future cash flows to be generated is zero. Likewise, for $F_B(V_B,K_B)$, and as K_B becomes infinitely large for any value of V_B, the option is also worthless. To respect this condition, we need to set constant $A_1 = 0$, leaving only constant $A_2 \neq 0$. Consequently, in our case the quadrant of interest is the second one - the one that corresponds to the pair of roots $\{\beta^+,\eta^-\}$. Thus, the general solution for the homogeneous part of the partial differential equation presented above, $F_B^H(V_B,K_B)$ is reduced and takes the following form:

$$F_B^H(V_B,K_B) \quad = \quad A_{10}V_B^{\beta^+}K_B^{\eta^-} \tag{A.9}$$

The particular solution for the non-homogeneous part of the partial differential equation, F_B^{NH} is given by the following equation:

$$F_B^{NH} \quad = \quad -\frac{c'}{r} \tag{A.10}$$

Joining the homogeneous and non-homogeneous solutions, we obtain the following expression, which satisfies the whole partial differential equation $(A.4)$:

$$F_B^H(V_B,K_B) + F_B^{NH} = F_B(V_B,K_B) = \quad A_{10}V_B^{\beta^+}K_B^{\eta^-} - \quad \frac{c'}{r} \tag{A.11}$$

The value matching condition implies that, when the trigger values for both V_B and K_B are simultaneously attained, the value of the option to invest, $F_B(V_B^*,K_B^*)$ must equal the project's expected NPV. Therefore:

$$F_B(V_B^*,K_B^*) \quad = \quad A_{10}V_B^{*\beta^+}K_B^{*\eta^-} - \frac{c'}{r} = V_B^* - K_B^* \tag{A.12}$$

Since this value-matching condition does not support homogeneity of degree one on both sides of the equation (which means that $\beta^+ + \eta^- \neq 1$), we follow Adkins and Paxson (2011) approach and construct a set of three simultaneous equations. The first is the elliptical equation $(A.8)$ and the other two equations are the two smooth-pasting conditions derived from the value-matching condition given by equation $(A.12)$.

The first smooth-pasting condition is the first-order derivative of the value-matching condition given by equation $(A.12)$, in respect to V_B^*. After some rearrangements, we reach the expression for the first smooth-pasting condition, given by equation $(A.13)$.

$$\beta^+(V_B^* - K_B^* + \frac{c'}{r}) \quad = \quad V_B^* \tag{A.13}$$

The second smooth-pasting condition is the first-order derivative of the same value matching condition, in respect to K_B^*. Likewise, after some rearrangements, we obtain the second smooth-pasting condition, given by equation $(A.14)$.

$$\eta^-(V_B^* - K_B^* + \frac{c'}{r}) \quad = \quad -K_B^* \tag{A.14}$$

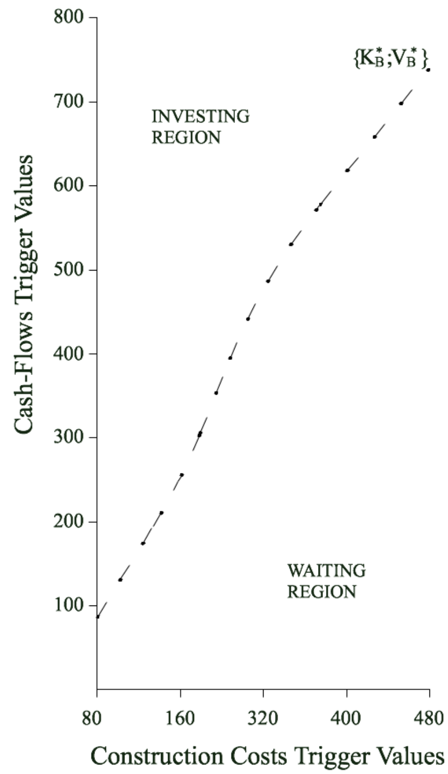
Thus, the set of three simultaneous equations is formed by equations $(A.8)$, $(A.13)$ and $(A.14)$. We need to set K_B^* since we are in the presence of four unknowns $(K_B^*, V_B^*, \beta^+, \eta^-)$ and only have three equations available. Being so, we need to fix K_B^* and solve the system of equations considering the value of the legal penalty included in the contract form, c'. By repeating this process, we determine the corresponding values for V_B^* and, hence, reach the discriminatory boundary that separates the investing region from the waiting region, composed by a set of pairs $\{K_B^*; V_B^*\}$.

Considering that the selected bidder investment opportunity is situated in the waiting region, the investment will be triggered when K_B^* and V_B^* are simultaneously attained, *i.e.*, $\{K_B, V_B\} \geq \{K_B^*, V_B^*\}$. When both triggers are simultaneously attained, the value of the option to invest equals the project's expected NPV. Otherwise, the selected bidder should wait and defer the investment. Hence, the value of the selected bidder investment opportunity is given as follows:

$$F_B(V_B, K_B) \quad = \quad \begin{cases} (V_B^* - K_B^* + \frac{c'}{r}) \left(\frac{V_B}{V_B^*}\right)^{\beta^+} \left(\frac{K_B}{K_B^*}\right)^{\eta^-} - \frac{c'}{r};\, if & \{K_B, V_B\} < \{K_B^*, V_B^*\} \\ V_B - K_B \qquad ;if & \{K_B, V_B\} \geq \{K_B^*, V_B^*\} \end{cases} \tag{A.15}$$

Figure A.1 shows the selected bidder discriminatory boundary, assuming that the value of the legal penalty actually included in the contract is $c' = 1.9222$. We set K_B^* ranging from 80 to 480 and consider that: $\sigma_{V_B} = 0.15$, $\sigma_{K_B} = 0.10$, $\rho_B = 0$, $r = 0.05$, $\delta_{B_V} = \delta_{B_K} = 0.03$ and $\gamma = 0.2$.

Figure A.1: the selected bidder discriminatory boundary



The discriminatory boundary included in Figure A.1 was determined assuming that the value of the contractual penalty, c' is the same, regardless of the amount considered for the selected bidder construction costs trigger. The discriminatory boundary is not a straight line due to the effect of the contractual penalty, $c' > 0$ in the system of simultaneous equations used to determine the set of pairs $\{K_B^*; V_B^*\}$.

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